

PHYSICS

DEPARTMENTAL WRITTEN EXAM

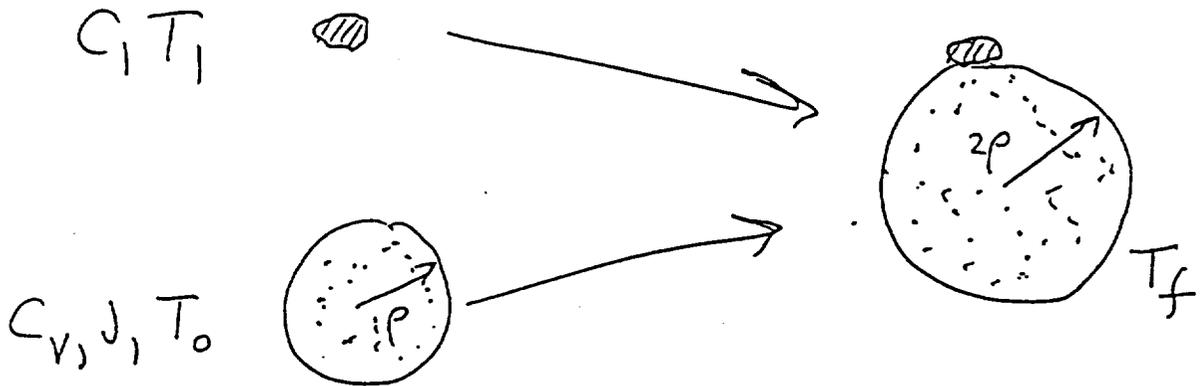
SOLUTIONS

FALL, 1999

PHYSICS DEPARTMENTAL EXAM – FALL 1999

SECTION 1:Problem 1.

The pressure, p , of a spherical bubble of radius ρ is given by $p = 2\sigma/\rho$, where the surface tension, σ , is constant. A bubble in a vacuum, filled with ν moles of ideal gas with heat capacity C_V in equilibrium at temperature T_0 . [You may assume that the heat capacity of the bubble wall is negligible.] Then a small particle with heat capacity C_1 at temperature T_1 comes in contact to the bubble, and the system slowly comes to a new equilibrium such that the radius of the bubble has increased by a factor of two.



- What is the final temperature of the system in terms of the initial temperature of the bubble, T_0 ?
- Calculate the total change in entropy of the particle and bubble.
- Calculate the initial temperature of the particle in terms of the parameters given and the gas constant R .

2.

$$(a) \quad pV = \mathcal{N}RT$$

$$\frac{\sigma}{2R} \frac{4}{3} \pi R^3 = \mathcal{N}RT$$

$$R^2 \propto T$$

$$\text{If } R \uparrow \times 2 \quad \underline{T_f = 4T_0}$$

$$(b) \quad \Delta S = \int \frac{dQ}{T}$$

$$\text{Particle} \quad \Delta S = \int_{T_0}^{T_f} \frac{C_p dT}{T} = C_p \ln \frac{T_f}{T_0}$$

$$\text{Bubble + gas} \quad \Delta S = \int_{T_0}^{4T_0} \frac{C_v dT}{T} + \int \frac{p dV}{T} = C_v \ln 4 + \mathcal{N}R \ln \frac{V_f}{V_i}$$

$$\mathcal{N}R \int \frac{dV}{V}$$

$$\frac{V_f}{V_i} = 8$$

$$\Delta S_{\text{total}} = C_p \ln \frac{4T_0}{T_0} + C_v \ln 4 + \mathcal{N}R \ln 8$$

2. $dQ = 0$

$$C_1(T_1 - T_f) = C_v(T_f - T_0) + \int p dV$$

$$\int p dV = \int_{p_i}^{p_f} \frac{V}{2p} 4\pi p^2 dp = 2\pi V \int_{p_i}^{p_f} p dp = \pi V p^2 \Big|_{p_i}^{p_f}$$

$$= 3\pi V p_i^2 = \left(\frac{9}{2}\right) \frac{4}{3} \pi p_i^3 \left(\frac{V}{2p_i}\right) = \frac{9}{2} p_i V_i = \frac{9}{2} J R T_0$$

$$T_1 = \frac{[4C_1 + 3C_v + \frac{9}{2}JR] T_0}{C_1}$$

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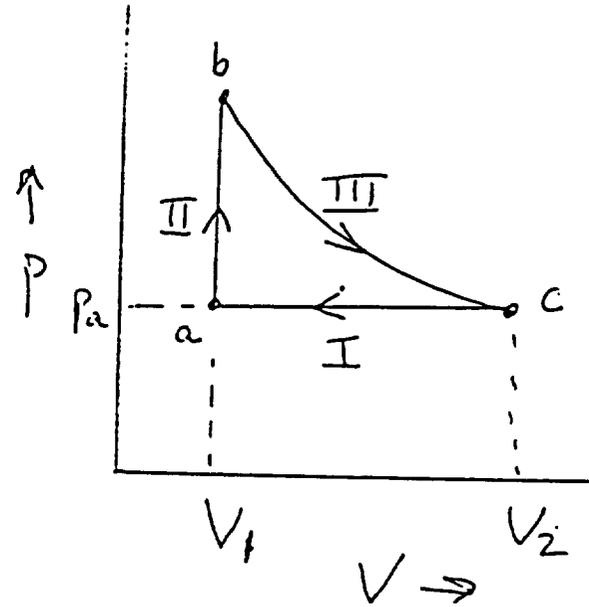
SECTION 1:

Problem 2.

Consider a photon gas is in equilibrium in a volume V . It is cycled quasistatically around the loop as shown in pressure-volume (p, V) space. Step III is adiabatic.

Use the relationships between the pressure, energy density, and the temperature of such a gas to answer the following questions.

[Note: If you recall these relationships, start there. If not, you'll need to develop them from more basic expressions.]



- What is the work, W , done, the change in internal energy, ΔE , and the heat, Q , added in step I?
- Develop the equation of state of the gas for an adiabatic transformation and use it to determine the factor that the temperature, T , of the gas changes as a result of step III? Indicate clearly the direction of this change (i.e., up or down).
- Calculate the change in entropy of the gas in step II, expressing your answer in terms of p_a , V_1 , V_2 , and the temperature of the gas, T_a , at (p_a, V_1) .

Internal energy density { e. }, $E = VU$

$$\underline{U \propto T^4}$$

$$\left. \begin{aligned} \text{e.}, \quad U &\propto \int_0^{\infty} \frac{\hbar^2 dk \hbar c k}{e^{\beta \hbar c k} - 1} \\ \text{making the integral dimensionless} \\ \Rightarrow U &\propto T^4 \end{aligned} \right\}$$

$$\underline{p = \frac{1}{3} U}$$

e.)

$$p = \sum_n \left(- \frac{\partial \epsilon_n}{\partial V} \right)$$

\hookrightarrow states of system

$$\epsilon_k = \hbar c k \propto \frac{1}{V}^{1/3} \Rightarrow p = \frac{1}{3V} \sum_n \epsilon_n = \frac{1}{3} U$$

$$\left\{ k = \left[\left(\frac{2\pi n_x}{L_x} \right)^2 + \left(\frac{2\pi n_y}{L_y} \right)^2 + \left(\frac{2\pi n_z}{L_z} \right)^2 \right]^{1/2} \sim \frac{1}{L} \sim \frac{1}{V}^{1/3} \right\}$$

a) Step I
 $p = \text{const.}$

$$W = p \Delta V = p_a (V_1 - V_2) < 0$$

$p = \text{const} = \frac{1}{3} U$ ~~so~~ const Work is done on the system

$$\Delta E = 3 p_a \Delta V$$

$$Q = \Delta E + W = \left(3 p_a \Delta V + p_a (V_1 - V_2) \right) = 4 p_a (V_1 - V_2) < 0$$

Heat flows out of the system

b) Step II

$$dQ = 0 = dE + dW$$

let $U = a T^4$ $p = \frac{1}{3} a T^4$

$$d(a V T^4) + p dV = 0$$

$$a T^4 dV + 4 a V T^3 dT + \frac{1}{3} a T^4 dV = 0$$

$$\frac{4}{3} a T^4 \frac{dV}{V} + \frac{4 a T^3 dT}{T} = 0$$

$$\frac{V^{\frac{1}{3}} T = \text{const}}{\textcircled{2}} \Rightarrow \frac{V^{\frac{1}{3}} p^{\frac{1}{4}} = \text{const}}{p V^{\frac{4}{3}} = \text{const}}$$

Step IV

Jan 10 F '99 2.3

$$V_1^{1/3} T_b = V_2^{1/3} T_c$$

So on step III

$$\frac{T_c}{T_b} = \left(\frac{V_1}{V_2} \right)^{1/3} < 1$$

Step II

(c) $n V = \text{const} \quad dW = 0$

$$dQ = dE = d(V_1 a T^4) = 4a V_1 T^3 dT$$

$$\Delta S = \int \frac{dQ}{T} = 4V_1 \int \frac{a T^3 dT}{T} = \frac{4}{3} V_1 a [T_b^3 - T_c^3]$$

Now on step I $p = \text{const} \Rightarrow T = \text{const}$

$$\Rightarrow T_a = T_c \quad \text{from (b) then } \left(\frac{T_a}{T_b} \right)^3 = \frac{V_1}{V_2}$$

$$\text{Thus } \Delta S_{II} = \frac{4}{3} V_1 a T_a^3 \left[\frac{V_2}{V_1} - 1 \right] = \frac{4 p_a (V_2 - V_1)}{T_a} > 0$$

Entropy increases.

$$\left\{ \begin{array}{l} \text{Note: } T = \text{const on step I.} \\ \text{so } \Delta S_I = \frac{Q_I}{T_a} = \frac{4 p_a (V_1 - V_2)}{T_a} = -\Delta S_{II} \quad \checkmark \checkmark \end{array} \right\}$$

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SECTION 2:

Problem 3.

Consider the harmonic oscillator Hamiltonian

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

and define

$$a = \sqrt{\frac{m\omega}{2\hbar}} \hat{x} + i\left(\frac{1}{2m\omega\hbar}\right)^{1/2} \hat{p}$$

- Find $[a, a^\dagger]$
- Show that $e^{-\lambda a^\dagger}|0\rangle$ (where $a|0\rangle = 0$) is an (unnormalized) eigenstate of a .
- Find the normalization constant for this state.
- Find the probability that a measurement of the energy will yield $\left(\frac{1}{2} + n\right)\hbar\omega$.

$$\begin{aligned}
 \text{a)} \quad & \sqrt{\frac{m\omega}{2\hbar}} \cdot \frac{i}{\sqrt{2m\omega\hbar}} \left(-[\hat{x}, \hat{p}] + [\hat{p}, \hat{x}] \right) \\
 & = \frac{i}{2\hbar} (-2i\hbar) = 1
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & a e^{\lambda a^\dagger} |0\rangle = [a, e^{\lambda a^\dagger}] |0\rangle \\
 & = [a, \sum_n \frac{\lambda^n (a^\dagger)^n}{n!}] |0\rangle \\
 & = \sum_{n=1} \lambda^n \frac{(a^\dagger)^{n-1}}{(n-1)!} |0\rangle = \lambda e^{\lambda a^\dagger} |0\rangle
 \end{aligned}$$

$$\begin{aligned}
 \text{c)} \quad & 1 = |c|^2 \langle 0 | e^{\lambda^* a} e^{\lambda a^\dagger} |0\rangle \\
 & = |c|^2 \sum_{n, n'} \langle 0 | a^n a^{\dagger n'} |0\rangle \frac{\lambda^{*n} \lambda^{n'}}{n! n'} \\
 & = |c|^2 \sum_n \langle 0 | a^n a^{\dagger n} |0\rangle \frac{|\lambda|^{2n}}{(n!)^2} \\
 & = |c|^2 \sum_n \frac{(|\lambda|^2)^n}{n!} \Rightarrow c = e^{-\frac{1}{2} |\lambda|^2}
 \end{aligned}$$

d) Since $H = \hbar\omega \left(\frac{1}{2} + a^\dagger a\right)$,
we have

$$e^{-\frac{1}{2}|\lambda|^2} e^{\lambda a^\dagger} |0\rangle$$

$$= \sum_n e^{-\frac{1}{2}|\lambda|^2} \frac{\lambda^n}{n!} (a^\dagger)^n |0\rangle$$

(note
 $a^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle$)

$$= \sum_n \frac{e^{-\frac{1}{2}|\lambda|^2} \lambda^n}{\sqrt{n!}} |n\rangle$$

\Rightarrow probability of n is $\frac{e^{-|\lambda|^2} |\lambda|^{2n}}{n!}$

(note : $\sum p_n = 1$)

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SECTION 2

PROBLEM 4.

Consider a spin- $\frac{1}{2}$ particle with magnetic moment $\frac{e\hbar}{2mc}$ in a uniform external magnetic field $\vec{B} = B\hat{z}$. The Hamiltonian of the system is

$$H = \vec{\mu} \cdot \vec{B} = -\left(\frac{eB}{mc}\right) S_z.$$

- (a) What are the energy eigenstates and eigenvalues of this Hamiltonian?
- (b) Consider a general initial state vector which can be written as a linear combination of energy eigenstates. What is the state vector after a time t ?
- (c) What is the expectation value of S_z at time t ?
- (d) What are the expectation values of S_x and S_y at time t ?
- (e) What do these expectation values in parts c) and d) reduce to if the initial wavefunction is measured to have spin $+\hbar/2$ along the \hat{x} direction?

(1) Consider a spin- $\frac{1}{2}$ particle with magnetic moment $\frac{e\hbar}{2mc}$ in a uniform external magnetic field $\vec{B} = B\hat{z}$. The Hamiltonian of the system is

$$H = -\vec{\mu} \cdot \vec{B} = -\left(\frac{eB}{mc}\right) S_z .$$

(a) What are the energy eigenstates and eigenvalues of this Hamiltonian?

$|\pm\rangle$, with $S_z = \pm\hbar/2$ and $E_{\pm} = \mp\frac{e\hbar B}{2mc} = \mp\hbar\omega/2$

(b) Consider a general initial state vector which can be written as a linear combination of energy eigenstates. What is the state vector after a time t ?

$$|\psi(0)\rangle = c_+ |+\rangle + c_- |-\rangle$$
$$|\psi(t)\rangle = c_+ e^{i\omega t/2} |+\rangle + c_- e^{-i\omega t/2} |-\rangle$$

(c) What is the expectation value of S_z at time t ?

$$\langle\psi(t)| S_z |\psi(t)\rangle = \frac{\hbar}{2} (|c_+|^2 - |c_-|^2)$$

(d) What are the expectation values of S_x and S_y at time t ?

$$S_x = \frac{\hbar}{2}\sigma_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$S_y = \frac{\hbar}{2}\sigma_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\langle\psi(t)| S_x |\psi(t)\rangle = \frac{\hbar}{2} (c_+^* c_- e^{-i\omega t} + c_-^* c_+ e^{i\omega t})$$

$$\langle\psi(t)| S_y |\psi(t)\rangle = \frac{\hbar}{2} (-i c_+^* c_- e^{-i\omega t} + i c_-^* c_+ e^{i\omega t})$$

(e) What do these expectation values in parts (c) and (d) reduce to if the initial wavefunction is measured to have spin $+\hbar/2$ along the \hat{x} direction?

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} (|+\rangle + |-\rangle)$$
$$c_+ = c_- = \frac{1}{\sqrt{2}}$$

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PART I

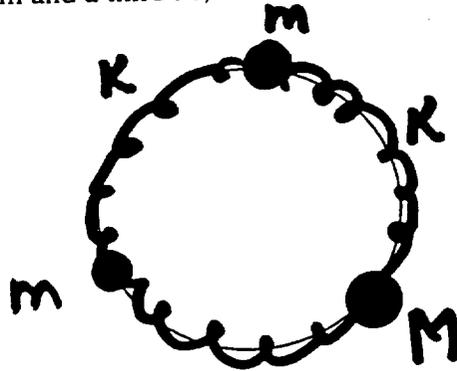
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SECTION 3:

PROBLEM 5:

Consider two masses m and a third M , as shown, connected by 3 springs k and constrained to move along the ring.

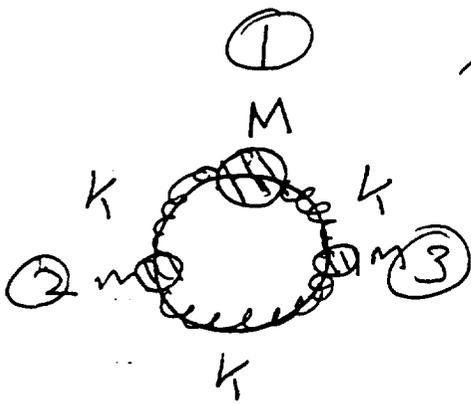


Calculate all mode frequencies and mode shapes (i.e. eigen values and eigen vectors).

Solution

Solution 5

26.



$$M\theta_1 + m\theta_2 + m\theta_3 = 0$$

$$\theta_1 = -\frac{m}{M}(\theta_2 + \theta_3) \equiv -\alpha(\theta_2 + \theta_3)$$

$$L = \frac{1}{2}(M\dot{\theta}_1^2 + m\dot{\theta}_2^2 + m\dot{\theta}_3^2) - \frac{1}{2}k[(\theta_3 - \theta_1)^2 + (\theta_2 - \theta_3)^2 + (\theta_1 - \theta_2)^2]$$

$$= \frac{1}{2}\left[M\alpha^2(\dot{\theta}_2 + \dot{\theta}_3)^2 + m(\dot{\theta}_2^2 + \dot{\theta}_3^2)\right]$$

$$- \frac{1}{2}k[(\theta_3 - \alpha(\theta_2 + \theta_3))^2 + (\theta_2 - \theta_3)^2 + (\alpha(\theta_2 + \theta_3) - \theta_2)^2]$$

$$L = \frac{1}{2}\left[(m + M\alpha^2)(\dot{\theta}_2^2 + \dot{\theta}_3^2) + 2M\alpha^2\dot{\theta}_2\dot{\theta}_3\right]$$

$$- \frac{k}{2}\left[(1 - \alpha)^2\theta_3^2 - 2\alpha(1 - \alpha)\theta_2\theta_3 + \alpha^2\theta_2^2 + \theta_2^2 - 2\theta_2\theta_3 + \theta_3^2\right]$$

$$(1 - \alpha)^2\theta_2^2 - 2(1 - \alpha)\alpha\theta_2\theta_3 + \alpha^2\theta_3^2$$

$$L = \frac{1}{2}\left[(m + M\alpha^2)(\dot{\theta}_2^2 + \dot{\theta}_3^2) + 2M\alpha^2\dot{\theta}_2\dot{\theta}_3\right]$$

$$- \frac{1}{2}k\left[(1 - 2\alpha + \alpha^2) + 1 + \alpha^2\right]\theta_2^2 + (1 - 2\alpha + \alpha^2 + 1 + \alpha^2)\theta_3^2$$

$$- 2(2\alpha(1 - \alpha) + 1)\theta_2\theta_3$$

2c.

we define

$$\begin{aligned} 2 + 2\alpha^2 - 2\alpha &\equiv c \\ 1 + 2\alpha - 2\alpha^2 &\equiv d \\ (m + M\alpha^2) &\equiv ma \\ M\alpha^2 &\equiv mb \end{aligned}$$

$$L = \frac{1}{2} \left[m (a (\dot{\theta}_2^2 + \dot{\theta}_3^2) + 2b \dot{\theta}_2 \dot{\theta}_3) - k (c (\theta_2^2 + \theta_3^2) - 2d \theta_2 \theta_3) \right]$$

$$\frac{d}{dt} \left[m(a\dot{\theta}_2 + b\dot{\theta}_3) \right] + [ck\theta_2 - dk\theta_3] = 0$$

$$\frac{d}{dt} \left[m(\dot{\theta}_3 a + b\dot{\theta}_2) \right] + [ck\theta_3 - dk\theta_2] = 0$$

$$\omega_0^2 = k/m$$

$$a \ddot{\theta}_2 + b \ddot{\theta}_3 + \omega_0^2 [c\theta_2 - d\theta_3] = 0$$

$$b \ddot{\theta}_2 + a \ddot{\theta}_3 + \omega_0^2 [c\theta_3 - d\theta_2] = 0$$

add

$$(a+b)(\ddot{\theta}_2 + \ddot{\theta}_3) + \omega_0^2 (c-d)(\theta_2 + \theta_3) = 0$$

$$\omega^2 = \omega_0^2 \frac{(c-d)}{(a+b)} \quad ; \quad \underline{\theta} = (-2\alpha, 1, 1)$$

(sym \rightarrow low ω)

subtract

$$a(\ddot{\theta}_2 - \ddot{\theta}_3) + b(\ddot{\theta}_3 - \ddot{\theta}_2) + \omega_0^2 [c(\theta_2 - \theta_3) - d(\theta_3 - \theta_2)] = 0$$

$$(a-b)(\ddot{\theta}_2 - \ddot{\theta}_3) + \omega_0^2 (c+d)(\theta_2 - \theta_3) = 0$$

$$\omega^2 = \frac{(c+d)}{(a-b)} \omega_0^2, \quad \theta = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

(anti-symm \rightarrow higher ω)

\downarrow
heavy mass stationary.

and zero frequency mode (rotation)

$$\omega^2 = 0$$

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PART I

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SECTION 3:

PROBLEM 6:

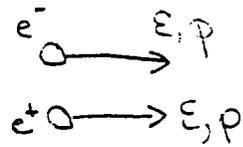
In interstellar space, gamma ray photons of sufficiently high energy ϵ_1 can interact with photons in the 3^0 K blackbody spectrum of energy ϵ_2 to produce electron-positron pairs. Find the minimum energy ϵ_1 for the gamma rays in order to allow pair creation.

~~PA Problem 3, Part I, Section 6~~

Exam problem

In interstellar space, gamma ray photons of sufficiently high energy ϵ_1 can interact with photons in the 3°K blackbody spectrum of energy ϵ_2 to produce electron-positron pairs. Find the minimum energy ϵ_1 for the gamma rays in order to allow pair creation.

Solution



zero kinetic energy in center of mass

$$\epsilon_1 = p_1 c, \quad \epsilon_2 = p_2 c, \quad \epsilon = \sqrt{p^2 c^2 + m^2 c^4}$$

energy consv. : $\epsilon_1 + \epsilon_2 = 2\epsilon$

momentum consv. : $p_1 - p_2 = 2p$

$$\Rightarrow \frac{\epsilon_1}{c} - \frac{\epsilon_2}{c} = 2p$$

$$\rightarrow (\epsilon_1 + \epsilon_2)^2 = 4(m^2 c^4 + p^2 c^2) = 4 \left(\frac{(\epsilon_1 - \epsilon_2)^2}{4} + m^2 c^4 \right)$$

$$\Rightarrow \boxed{\epsilon_1 \epsilon_2 = m^2 c^4}$$

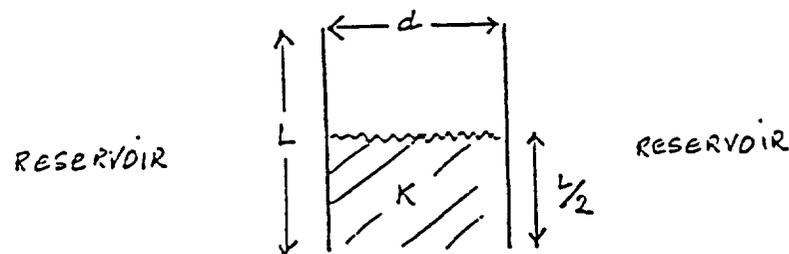
$$\Rightarrow \boxed{\epsilon_1 > m^2 c^4 / \epsilon_2}$$

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SECTION 4:

Problem 7.

A parallel-plate capacitor with square plates of side-length L and plate separation d is charged to a potential V and disconnected from the source of e.m.f. It is then vertically inserted into a large reservoir of dielectric liquid with relative dielectric constant κ and density ρ until the liquid half fills the space between the capacitor plates as shown.

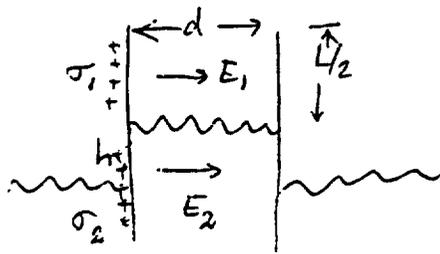


- i) What is the capacitance of the system?
- ii) What is the electric field strength between the capacitor plates?
- iii) What is the distribution of charge density over the plates?
- iv) What is the difference in vertical height between the level of liquid within the capacitor plates and that in the external reservoir?

Prob 7 Solution

initially: $C = \frac{A}{4\pi d}$, $Q = \frac{A}{4\pi d} V$, $\sigma = \frac{V}{4\pi d}$

After inserting in liquid:



$$E_1 = 4\pi \sigma_1$$

$$E_2 = \frac{4\pi \sigma_2}{\kappa}$$

The plate is an equipotential so $E_1 = E_2 = \frac{V'}{d}$

V' = final potential difference.

$$\therefore \sigma_2 = \kappa \sigma_1$$

total charge is conserved:

$$\sigma_2 \frac{A}{2} + \sigma_1 \frac{A}{2} = \sigma A \Rightarrow$$

$$\sigma_1 = \frac{2\sigma}{1+\kappa}$$

$$\sigma_2 = \frac{2\kappa\sigma}{1+\kappa}$$

Ans
(iii)

$$E_1 = E_2 = \frac{8\pi \sigma}{1+\kappa} = \frac{2}{1+\kappa} \frac{V}{d}$$

Ans (ii)

$$C = Q/V' = \frac{A}{4\pi d} \frac{V}{V'} = \frac{1+\kappa}{2} \frac{A}{4\pi d}$$

Ans i

(can also do this treating system as 2 capacitors in parallel)

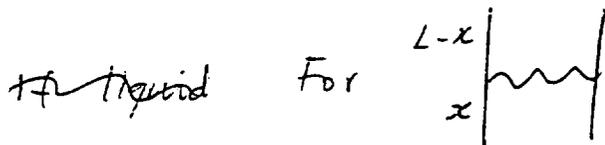
$$U = \frac{1}{2} \frac{Q^2}{C} + \text{rest } U_{\text{grav}}$$

volume of liquid above outside level

$$U_{\text{grav}} = \rho g \left(\frac{A d h}{L} \right) \frac{h}{2}$$

$$\delta U = 0 \Rightarrow \frac{Q^2}{2C^2} \delta C + \delta U_{\text{grav}} = 0$$

$$\Rightarrow \rho g \frac{A d h}{L} \frac{\delta h}{2} = \frac{Q^2}{2C^2} \delta C$$



$$C = \frac{A}{4\pi d} \frac{(L-x) + kx}{L}$$

$$\Rightarrow \delta C = \frac{A}{4\pi d} \cdot \frac{(k-1) \delta x}{L}$$

$$\Rightarrow \rho g \frac{A d h}{L} \frac{\delta h}{2} = \frac{Q^2}{2} \left(\frac{4\pi d}{A} \right)^2 \left(\frac{2}{1+k} \right)^2 \cdot \left(\frac{A}{4\pi d} \right)^2 (k-1) \frac{\delta x}{L}$$

$$\Rightarrow h = \frac{Q^2}{2} \left(\frac{2}{1+k} \right)^2 \frac{4\pi d}{A} (k-1) \frac{1}{\rho A d g}$$

$$h = \frac{V^2}{2} \left(\frac{2}{1+k} \right)^2 \frac{(k-1)}{4\pi \rho d^2 g}$$

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SECTION 5:

Problem 8.

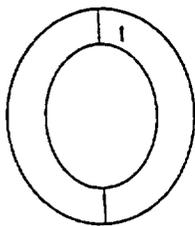
How thick must be the walls of a steel cylinder, of diameter 10 cm, if the inside is to be safely pressurized to 100 atmospheres? If you do not know the relevant parameter for steel, describe how you would estimate it and use the value that you obtain.

Possible questions for the qualifying exam from J. Goodkind.

1) How thick must be the walls of a steel cylinder, of diameter 10 cm, if the inside is to be safely pressurized to 100 atmospheres. If you do not know the relevant parameter for steel describe how you would estimate it and use the value that you obtain.

Answer: The total outward force on the cylinder is

$$F = P\pi R^2 l$$



Where P is the pressure inside the cylinder R is its inside radius, and l its length. This force is applied over the area of the walls of thickness t so that the force per unit area on the walls is

$$F/tl = P\pi R^2 l/tl = P\pi R^2 t$$

This must be less than the tensile strength of steel (or for complete safety, usually less than 1/4 the elastic limit) which is of the order of $2 \times 10^8 \text{ nt m}^{-2}$ or $4 \times 10^4 \text{ lb in}^{-2}$. 1 atmosphere = 10^5 Nt m^{-2} . Therefore the minimum t is

$$t = 10^7 \pi (0.05)^2 2 \times 10^8 = 1.5 \times 10^{-3} \text{ m} = 1.5 \text{ mm}$$

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SECTION 5:

Problem 9.

An instrument measures the following number of radiation events per hour, in order of increasing time: 65, 57, 61, 63, 59, 50, 51, 48, 46

- v) Is the measured quantity constant in time? Give a detailed, quantitative discussion.
- vi) Theory calculations lead us to expect that $A = M/k = 1.000$, where k is a constant, with a measured value of 45 ± 2 , and M is the measured mean rate per hour. Are the measurements consistent with this prediction?

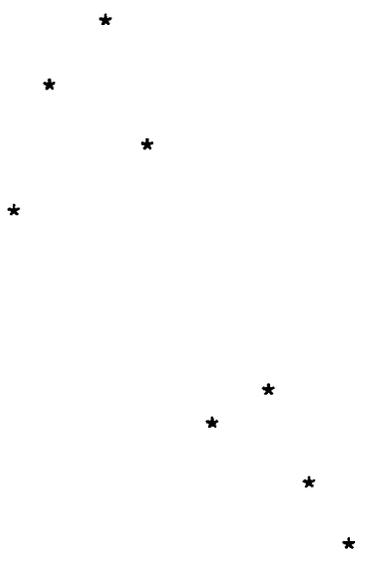
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Answer to a)

Should plot the data, which is the only reasonable way to answer these questions.

The radiation events should have a Poission distribution in the number of events per unit time. The means will be constant if the source is constant, the experimental setup (eg distance from source to detector) is constant, and the instrumental sensitivity is constant.

65 *
64
63
62
61 *
60
59 *
58
57 *
56
55
54
53
52
51
50
49
48
47
46
46



Looks like rate is declining.
Many ways to test if this is statistically significant.

1)
The rates should have a Poission distribution, which will be approximately Normal for these large values.
The distribution of rates appears double peaked, with a lack of values near the mean. It appears to be non-Normal, but a test is required to determine if this is significant.

2)
Mean is 55.55
standard deviation = 6.97
dispersion = standard deviation squared = 48.6

For a Poission distribution, standard deviation is mean squared.
The data are consistent with this, and hence the dispersion is consistent with a constant mean rate.

3)
Direct comparison with the null hypothesis of a constant mean requires a test for goodness of fit. The Chi-squared test is appropriate.
Under the null hypothesis, the rates will be Normally distributed about the mean, and the dispersions of the individual measurements will be about the measured values.

Then: $Chi_sq = \sum \text{over } i (x_i - \text{mean})^2 / \text{dispersion}$, or

$$\sum \text{over } i (x_i - \text{mean})^2/x_i$$

It is best to list the deviations of each measurement from the mean.

x_i	= 65	57	61	63	59	50	51	48	46
$(x_i - \text{mean})^2/x_i$	= 1.37	0.04	0.49	0.88	0.20	0.62	0.41	1.19	1.98

None of the values are more than 2 sigma from the mean, which is consistent with a Normal distribution with constant mean. In a sample of 20, would expect one which was more than 2 sigma from mean.

Chi-squared = 7.18.

There are 8 degrees of freedom, hence expect chi-sq to be about 8, which it is. Hence distribution of data about the mean is consistent with a constant mean.

4) Correlation test.

There remains the impression that the data are correlated, with the mean value decreasing with rank in the time sequence. A rank correlation test is appropriate. Some discussion of this, but not the test itself, is required.

The test result would probably be null, since the above tests showed that the dispersion is consistent with a constant mean.

However, data analysis must be guided by knowledge of the experiment. If there is a physical reason to expect a change in rate (eg decay of a radioactive source), then it is appropriate to ask more about the experiment, and to fit the data to estimate the rate of change. The function to be fit will depend on the experiment. eg: a radioactive decay might be fit with an exponential plus a constant background term.

We do not know the times of the observations, and whether data is continuous and equally spaced. There might be a large gap in time between first five and last four measurements.

Summary: there is no statistical evidence for a change, but there is a hint of a decline in time, which should be discussed.

Answer to b)

The mean rate is 55.55, and the uncertainty can be estimated in two ways:

Total of 500 events were recorded in 9 hours.
The error in this is $\sqrt{500}/9$

$$M = 55.55 \pm 2.48$$

A less accurate, but acceptable estimate for the error on the mean can be obtained assuming that the data are Normally distributed:

$$\text{error} = (\text{standard deviation})/\sqrt{\text{number of measurements}} = 6.97/3 = 2.32$$

$$A = M/k = 55.55/45 = 1.234 \pm 0.078$$

where we use propagation of errors to get standard deviation of A:

$$[s(A)/A]^2 = [s(M)/M]^2 + [s(k)/k]^2$$

where $k=45$, $s(k)$ = standard deviation of $k = 2$,

$$s(A) = 0.078$$

The estimated A is $(1.234-1.000)/0.078 = 3.0$ standard deviations away from the prediction, so the data are not consistent.

SECTION 1:

Problem 10.

Two classical spin $-\frac{1}{2}$ particles are free to move on a ring. The Hamiltonian for this system is

$$H = J(\vartheta)\sigma_1\sigma_2 - \mu H(\sigma_1 + \sigma_2) + U(\vartheta) \quad ,$$

where ϑ is the smallest relative angle between the two particles, ranging from 0 to π . The potential is

$$U(\vartheta) = \begin{cases} +\infty & \text{if } 0 \leq \vartheta < \alpha \\ 0 & \text{if } \alpha \leq \vartheta \leq \pi \end{cases} \quad J(\vartheta) = \begin{cases} J_0 & \text{if } 0 \leq \vartheta < \beta \\ 0 & \text{if } \beta \leq \vartheta \leq \pi \end{cases} \quad ,$$

with $0 < \alpha < \beta < \pi$. Thus, the particles experience an infinite "hard core" repulsion whenever the relative angle is smaller than α . In addition, they interact with a short-ranged antiferromagnetic coupling J_0 whenever the relative angle is smaller than β . The external magnetic field is H and the magnetic moment of each particle is μ . For this problem, take $\alpha = \frac{1}{4}\pi$ and $\beta = \frac{1}{2}\pi$. Don't confuse the angle β with $1/k_B T$!

Compute the zero-field magnetic susceptibility $\chi(T) = (\partial M / \partial H)|_{H=0}$ as a function of temperature. Recall that the magnetization is given by $M = \mu \sum_i \langle \sigma_i \rangle$.

Solution

Let $\theta_2 = \theta_1 + \phi$. Then the partition function is

$$Z(T, H) = \sum_{\sigma_1, \sigma_2} \int_0^{2\pi} d\theta_1 \left\{ 2 \int_{\alpha}^{\beta} d\phi e^{-J_0 \sigma_1 \sigma_2 / k_B T} e^{\mu H (\sigma_1 + \sigma_2) / k_B T} + 2 \int_{\beta}^{\pi} d\phi e^{\mu H (\sigma_1 + \sigma_2) / k_B T} \right\}$$

$$= 4\pi(\beta - \alpha) \left(e^{-J_0 / k_B T} \cdot 2 \cosh(2\mu H / k_B T) + 2 e^{J_0 / k_B T} \right) + 4\pi(\pi - \beta) \left(2 \cosh(2\mu H / k_B T) + 2 \right).$$

The magnetization is

$$M(T, H) = -\frac{\partial F}{\partial H} = \frac{\left((\beta - \alpha) e^{-J_0 / k_B T} + (\pi - \beta) \right) 2\mu \sinh(2\mu H / k_B T)}{(\beta - \alpha) \left(e^{-J_0 / k_B T} \cosh(2\mu H / k_B T) + e^{J_0 / k_B T} \right) + (\pi - \beta) \left(\cosh(2\mu H / k_B T) + 1 \right)}$$

$$= \frac{(\beta - \alpha) e^{-J_0 / k_B T} + (\pi - \beta)}{(\beta - \alpha) \cosh(J_0 / k_B T) + (\pi - \beta)} \cdot \frac{2\mu^2 H}{k_B T} + \mathcal{O}(H^2).$$

Hence, the magnetic susceptibility is

$$\chi(T) = \frac{\partial M}{\partial H} \Big|_{H=0} = \frac{2\mu^2}{k_B T} \cdot \frac{(\beta - \alpha) \exp(-J_0 / k_B T) + (\pi - \beta)}{(\beta - \alpha) \cosh(J_0 / k_B T) + (\pi - \beta)}$$

$$= \frac{2\mu^2}{k_B T} \cdot \frac{s + \exp(-J_0 / k_B T)}{s + \cosh(J_0 / k_B T)},$$

where $s \equiv (\pi - \beta) / (\beta - \alpha)$. With $\alpha = \frac{1}{4}\pi$ and $\beta = \frac{1}{2}\pi$, we have $s = 2$.

Examining the above expression for $\chi(T)$ in various limits, we note that as $T \rightarrow 0$ we have $\chi \sim (4\mu^2 / k_B T) \exp(-2J_0 / k_B T) \rightarrow 0$. This is because the particles are in their lowest energy state, which requires that they be antiferromagnetically aligned (and within an angle β of each other). It requires an energy $2J_0$ to change the antiferromagnetic configurations $\uparrow\downarrow / \downarrow\uparrow$ to a ferromagnetic one $\uparrow\uparrow / \downarrow\downarrow$. Another familiar limit occurs when $s \rightarrow \infty$, *i.e.* $\beta \rightarrow \alpha$. In this case, there is no longer any antiferromagnetic interaction to speak of (one can also take $J_0 \rightarrow 0$ to achieve the same result). The angular degrees of freedom then are simply irrelevant to the magnetism, and the susceptibility is then $\chi = N\mu^2 / k_B T$, the familiar noninteracting result for N particles, with $N = 2$.

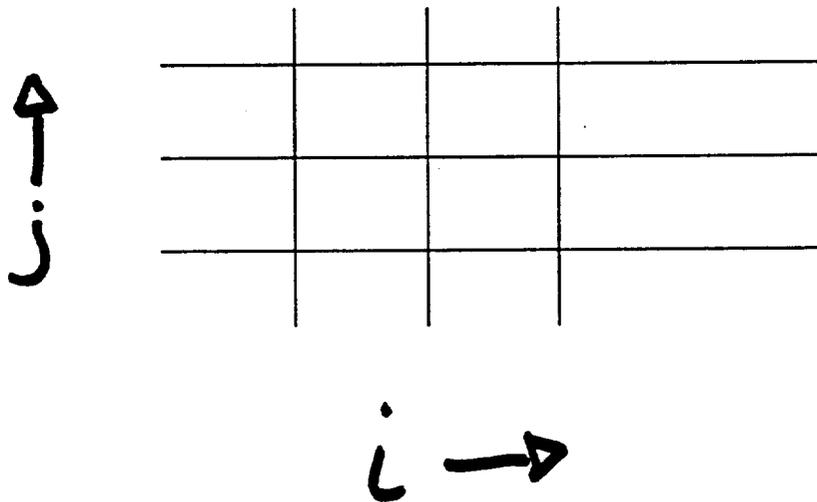
SECTION 2:

Problem 11.

Consider a two-dimensional square array of $2N$ massless strings of unperturbed length $(N+1)a$ with fixed endpoints. The strings are stretched to a tension T and have point masses m located at each intersection ia, ja , where $i < N, j < N$.

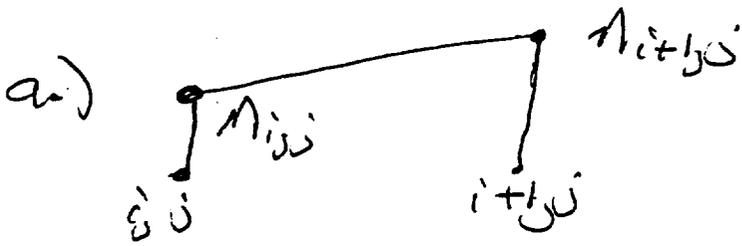
a) Construct the equations of motion and Lagrangian for the small transverse displacements X_{ij} of the masses. Find the exact dispersion relation $\omega(k)$ for traveling waves with wave vector $k = k_x \hat{x} + k_y \hat{y}$.

b) In the limit of a continuous mass distribution, show that this system obeys the two-dimensional wave equation. What is the corresponding dispersion relation for traveling waves?



~~140~~ 1.

Solution



$$dl = \left(a^2 + (m_{i,j} - m_{i+1,j})^2 \right)^{1/2} - a$$

$$\approx \frac{1}{2a} (m_{i,j} - m_{i+1,j})^2$$

$$L = \frac{1}{2} \sum_{i,j} m \dot{m}_{i,j}^2 - \sum_{i,j} \frac{\tilde{T}}{2a} \left[(m_{i,j} - m_{i+1,j})^2 + (m_{i,j} - m_{i,j+1})^2 \right]$$

is Lagrangian

$$\Rightarrow m \ddot{m}_{i,j} + \frac{\tilde{T}}{a} \left[4m_{i,j} - (m_{i+1,j} + m_{i-1,j}) - (m_{i,j+1} + m_{i,j-1}) \right] = 0$$

Look for solutions:

is Egn. Motion

$$m_{i,j} = e^{-i\omega t} e^{i(k_x i a + k_y j a)}$$

$$\Rightarrow \omega^2 = \frac{2\tilde{T}}{am} \left[2 - \cos k_x a - \cos k_y a \right]$$

is dispersion relation.

b.) Can write equation of motion as:

$$\ddot{M}_{ij} + \frac{\tilde{\gamma}/a}{m/a^2} \left[\frac{2M_{ij} - M_{i+1j} - M_{i-1j}}{a^2} + \frac{2M_{ij} - M_{ij+1} - M_{ij-1}}{a^2} \right] = 0$$

as $\left\{ \begin{array}{l} m/a^2 \rightarrow \rho \\ a \rightarrow 0 \end{array} \right.$ $\left\{ \begin{array}{l} \tilde{\gamma} \rightarrow \gamma \\ \text{mass/area} \\ \text{surface tension} \end{array} \right.$

$$\Rightarrow \frac{\partial^2 \eta}{\partial t^2} - \frac{\gamma}{\rho} \left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right) = 0$$

dispersion relation is: $\left\{ \begin{array}{l} \omega^2 = k^2 v^2 \\ v^2 = \gamma/\rho \end{array} \right.$

SECTION 2Problem 12.

What fraction of the acoustic energy would be reflected from an interface between water and glass? The boundary conditions for an acoustic wave are that the particle displacement and the pressure must be continuous across a boundary between two media. If you do not know the velocity of sound and the density of the two materials, make a guess based on some knowledge of other materials and use the values you obtain to solve the problem. Assume that the waves are normally incident to the interface.

3) What fraction of the acoustic energy would be reflected from an interface between water and glass. The boundary conditions for an acoustic wave are that the particle displacement and the pressure must be continuous across a boundary between two media. If you do not know the velocity of sound and the density of the two materials make a guess based on some knowledge of other materials and use the values you obtain to solve the problem. Assume that the waves are normally incident to the interface.

Answer: Assume that the particle displacement is described as a wave traveling in the x direction

$$s = s_0 \exp(i(kx - \omega t))$$

The pressure is then given by

$$p = -B \frac{ds}{dx} \cdot c^2 = \frac{B}{\rho} \cdot c = \frac{\omega}{k}$$

where B is the bulk modulus so that with incident and reflected waves in medium 1 and a transmitted wave in medium 2, the boundary conditions lead to the equations

$$s_i + s_r = s_t$$

$$-c_1 \rho_1 s_i + c_1 \rho_1 s_r = -c_2 \rho_2 s_t$$

$$s_r = \frac{c_2 \rho_2 - c_1 \rho_1}{c_2 \rho_2 + c_1 \rho_1} s_i$$

The speed of sound in glass is 5500 m/sec (most solids are about 5000 m/sec). The speed in water is 1460 m/sec (most fluids will have lower velocity than most solids). The density of water is 1 g/cc and the density of glass is 2.6 g/cc (greater than water since it does not float). The power in an acoustic wave is proportional to the square of the particle displacement. Thus the reflected power is 0.66 times the incident power.

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PART II

Score _____

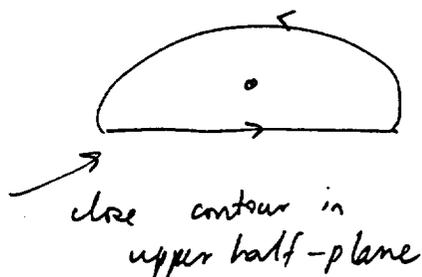
SECTION 3

Problem 13

Evaluate: $I = \int_0^{\infty} \frac{dx}{1+x^2+x^4}$

Evaluate: $I = \int_0^{\infty} \frac{dx}{1+x^2+x^4}$

Solution: $I = \frac{1}{2} \int_{-\infty}^{\infty} \frac{dz}{1+z^2+z^4}$



Poles at $1+z^2+z^4 = 0$

let $z^2 = z$, poles at $1+z+z^2 = 0$

ie. $z = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$

The roots are the cube roots of unity,

$$z = e^{2\pi i/3}, e^{4\pi i/3}$$

$$\therefore x = \sqrt{z} = \pm e^{\pi i/3}, \pm e^{2\pi i/3}$$

Poles in upper half-plane: $x = e^{\pi i/3}, e^{2\pi i/3}$

$$I = \frac{1}{2} \cdot (2\pi i) \sum (\text{residues in upper half-plane})$$

$$= \pi i \left[\left(\frac{1}{2x+4x^3} \right) \Big|_{x=e^{\pi i/3}, e^{2\pi i/3}} \right]$$

$$= \pi i \left(-\frac{i}{2\sqrt{3}} \right) = \frac{\pi}{2\sqrt{3}}$$

SECTION 3Problem 14.

Consider the differential equation

$$0 = \frac{d^2}{dx^2} (xP) - \alpha \frac{dP}{dx} - \beta P$$

- a) Find the general form of the solution (including two unknown constants) near $x = 0$.
- b) Do the same for $x \rightarrow \infty$.

(Note: Make sure you give answers for all possible values of α)

GRAD

Math method

Consider the differential equation

$$0 = \frac{d^2}{dx^2} (xP) - \alpha \frac{dP}{dx} - \beta P$$

- a) Find the general form of the solution (including two unknown constants) when $x=0$
- b) Do the same for $x \rightarrow \infty$

(note: make sure you give answer for all possible values of α)

a) Assuming fP is negligible gives

$$\frac{d^2}{dx^2} (xP) = \alpha \frac{dP}{dx}$$

$$P \sim x^\gamma \quad (\gamma+1)\gamma = \alpha\gamma$$

$$\gamma = \alpha - 1$$

so $P \sim C_1 x^{\alpha-1}$ (if $\alpha \neq 1$, $P \sim C_2 \ln x$)

Constant solution requires inclusion of fP ; as
other case is

$$P \sim C_2 + Dx$$

$$(2D - \alpha D) = \beta C_2 \quad P \sim C_2 \left(1 + \frac{f x}{\alpha - \alpha} \right)$$

$$D = \frac{f}{2 - \alpha} C_2$$

(if $\alpha = 2$, $P \sim C_2 (1 + \beta x \ln x)$)

b) Near ∞ , neglect $-\alpha \frac{dP}{dx}$ term

$$\frac{d^2}{dx^2} (xP) = \beta P \quad \frac{3}{x} (xP)$$

$$P \sim C_{\pm} / x e^{\pm 2\sqrt{\beta x}}$$

SECTION 4:

Problem 15.

Consider a liquid conductor with a surface charge σ_0 .

- a) First consider the liquid with a flat surface. Find the electric field and the pressure (due to electrostatics) on the surface. Is the pressure positive (i.e. pushing on the liquid) or negative?
- b) Now imagine the surface becomes slightly corrugated, being described by

$$z(x) = \delta \cos kx \quad \delta \ll 1$$

Find the correction to the electric potential above the surface.

(hint: if $\phi = \phi_0(z) + \delta\phi$, $\bar{\nabla}^2 \delta\phi = 0$ and the surface must still be an equipotential)

- c) Is the pressure change positive or negative in the outwardly protruding (i.e. $\cos kx > 0$) parts of the surface.

0) For a charged planar surface,

$$\phi_0(z) = -4\pi\sigma_0 z \quad z > 0$$

$$\vec{E} = 4\pi\sigma_0 \hat{z} \quad ; \text{ pressure} = \frac{E^2}{8\pi} = 2\pi\sigma_0^2 \quad (1)$$

b) $\delta\phi$ must vary as $\cos Kx$ along the surface.
The only solution of Laplace's equation is
this is $-Kz$

$$\delta\phi = \phi_1 \cos Kx e^{-Kz}$$

At dielectric surface

$$\phi(z) = 0 \approx \frac{\partial\phi_0}{\partial z} (\delta \cos Kx) + \phi_1 \cos Kx$$

$$\Rightarrow \phi_1 = +\delta (4\pi\sigma)$$

$$\vec{E} = -\vec{\nabla}\phi \quad \phi = -4\pi\sigma_0 z - Kz + 4\pi\sigma_0 \delta (\cos Kx e^{-Kz})$$

c) At positive $\cos Kx$, change in the electric field is $4\pi\sigma K > 0$. This corresponds to a higher value of the (outward) pressure. (Note: this is an unstable situation - the more the bump, the more the pulling pressure)

SECTION 5

Problem 16.

Consider a beam of neutrons of mass m , momentum p , magnetic moment μ_n , and polarization P defined by

$$P = \frac{n \uparrow - n \downarrow}{n \uparrow + n \downarrow}$$

where $n \uparrow$ and $n \downarrow$ are the numbers of neutrons with spins up and down, respectively. An unpolarized beam is incident from $x = -\infty$ upon a homogeneous magnetic field \mathbf{B} at $x > 0$. At $x < 0$ the magnetic field is zero. Taking into account the spin splitting in the magnetic field calculate the polarization of the reflected beam.

SOLUTION By Aneesh MANOHAR

SCORE _____

PHYSICS DEPARTMENTAL EXAMINATION #16

Student Identification # _____

$$H_{int} = - \mu \vec{\sigma} \cdot \vec{B} \quad V$$



The spin \uparrow (along B) state has $E = -\mu B$
 \downarrow $E = \mu B$

For a neutron $\mu_n < 0$ so $\text{spin } \uparrow = +|\mu B|$
 $\downarrow = -|\mu B|$ } $V = |\mu B| S_z$

[this is not an important point, if $\mu_n > 0$, then $\uparrow \leftrightarrow \downarrow$ and $P \rightarrow -P$]

Incident beam has momentum p , energy $E = \frac{p^2}{2m}$ ($v < c$).

Assume $E > |\mu B|$. Then

$$x < 0 \quad \psi = e^{ipx/\hbar} + R e^{-ipx/\hbar}$$

$$x > 0 \quad \psi = T e^{ikx/\hbar}, \quad V + \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m}$$

$$\Rightarrow k(x) = \sqrt{p^2 - 2m|\mu B| S_z} \quad S_z = \pm \frac{\hbar}{2}$$

ψ continuous

$$1 + R = T$$

ψ' continuous

$$p(1-R) = \hbar k T$$

\Rightarrow

$$R = \frac{p - \hbar k}{p + \hbar k}$$

$$T = \frac{2p}{p + \hbar k}$$

Note: If you use additional sheets for this problem, insert ID number, problem number, and make sure you number all the pages; then staple them together.

PHYSICS DEPARTMENTAL EXAMINATION

Student Identification # _____

Reflection probability = $|R|^2 = \left| \frac{p-k}{p+k} \right|^2 = w$

Incident flux $\uparrow = I$
 $\downarrow = I$

Reflected $\uparrow = I w_r$
 $\downarrow = I w_d$

Polarization = $\frac{I w_r - I w_d}{I w_r + I w_d} = \frac{w_r - w_d}{w_r + w_d}$

$$= \frac{\left| \frac{p - k(+)}{p + k(+)} \right|^2 - \left| \frac{p - k(-)}{p + k(-)} \right|^2}{\left| \frac{p - k(+)}{p + k(+)} \right|^2 + \left| \frac{p - k(-)}{p + k(-)} \right|^2}$$

$$= \frac{|p - k(+)|^2 |p + k(-)|^2 - |p + k(+)|^2 |p - k(-)|^2}{\dots}$$

$$= \frac{2p [k(-) - k(+)] [p^2 - k(+k(-))]}{[p^2 + k(+)^2][p^2 + k(-)^2] - 4p^2 k(+k(-))}$$

if $E < |AB|$ then \uparrow is reflected, ie $k(+)$ is imaginary so $w_r = 1$

$$P = \frac{4p k(-)}{p^2 + [k(-)]^2} = \frac{2p \sqrt{p^2 + |AB| 2m}}{p^2 + m|AB|}$$

Note: If you use additional sheets for this problem, insert ID number, problem number, and make sure you number all the pages; then staple them together.

ID Number _____

PART II

Score _____

SECTION 5:

Problem 17

Calculate the Born Approximation differential cross-section $d\sigma/d\Omega$ for scattering from a target, where the interaction potential between the incident and target atoms is $V(r) = V_0 e^{-r/a}$

Consider:

- a) The target is a single atom at $r = 0$.
- b) The target is two atoms, at $\vec{r} = \pm \frac{d}{2} \hat{z}$.

In both cases, the beam is incident along the z -direction.

In the Born approximation,

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{4\pi^2 \hbar^4} |V(\mathbf{q})|^2$$

$$V(\mathbf{q}) = \int d^3r V(r) e^{i\mathbf{q} \cdot \mathbf{r}}$$

For a) $V(r) = V_0 e^{-r/a}$

$$V_1(\mathbf{q}) = 2\pi V_0 \int_0^\infty e^{-r/a} \frac{2 \sin q r}{q r} r^2 dr$$

$$= (4\pi V_0) \frac{4/a}{(q^2 + 1/a^2)^2}$$

b) Now $V(\mathbf{r}) = V(\mathbf{r} - \frac{d\hat{z}}{2}) + V(\mathbf{r} + \frac{d\hat{z}}{2})$

$$V_2(\mathbf{q}) = \int d^3r \left(e^{i\mathbf{q} \cdot \frac{d\hat{z}}{2}} + e^{-i\mathbf{q} \cdot \frac{d\hat{z}}{2}} \right) e^{i\mathbf{q} \cdot \mathbf{r}} V(r)$$

(This used the fact that the dipole is oriented along the z axis). This gives

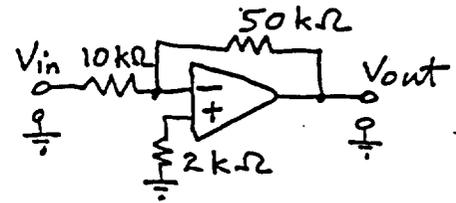
$$V_2(\mathbf{q}) = 2 \cos \frac{q_z d}{2} V_1(\mathbf{q})$$

SECTION 6

Problem 18.

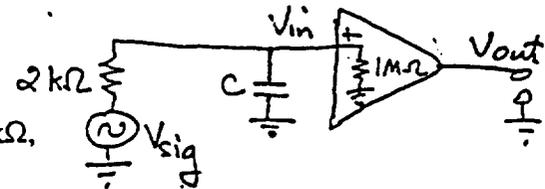
An ideal Operational Amplifier is configured as shown.

- (2 pts) a) What is the gain (V_{OUT}/V_{in}) of the circuit at a frequency of 1 kHz?



- (2 pts) b) What is the input impedance of the circuit at 1 kHz?

An instrumentation amplifier with gain 30 ($V_{out}/V_{in} = 30$) from DC to 1 MHz and input impedance $R_{in} = 1M\Omega$ is connected to a signal source with output impedance $R_{out} = 2k\Omega$, using a cable with capacitance $C = 2000$ pF.



- c) (2 pt) What is the signal gain (V_{out}/V_{sig}) for a signal at $f = 500$ kHz?

A noise-free time-varying signal $-3 \leq V_{sig} \leq 3$ Volts is recorded using a 12-bit waveform digitizer with range $-10 \leq V_{in} \leq 10$ Volts, clocked at 10^5 samples/sec.

- d) (2 pt) What is the signal-to-noise ratio for the recorded data?

- e) (2 pt) What filtration must be applied before the digitizer input to prevent "aliasing" of the input signal frequencies?

