Thomson Scattering - free e^-'s

assume NR - $\frac{|\frac{\vec{v}}{c}\vec{B}|}{|\vec{E}|} = |\frac{\vec{v}}{c}| << 1$

$$m\ddot{\vec{r}} = -e\vec{E} = Re\{-e\vec{E}_0 e^{-i(\vec{k}\cdot\vec{r}-\omega t)}\}$$

$\vec{r} \to \vec{r} + \Delta\vec{r}(t)$, $|\Delta\vec{r}|k = \frac{\omega}{c}|\Delta\vec{r}| = |\frac{v}{c}| << 1$ → neglect

Δr in exponential

$$\Delta\vec{r} = Re\left\{\frac{e\vec{E}}{m\omega^2} e^{i\vec{k}\cdot\vec{r}-i\omega t}\right\}$$

$$\vec{p}(t) = -e\Delta\vec{r} = Re\left\{-\frac{e\vec{E}}{m\omega^2} e^{i\vec{k}\cdot\vec{r}-i\omega t}\right\} = Re\{\vec{p}\, e^{-i\omega t}\}$$

$$\vec{p} = -\frac{e\vec{E}}{m\omega^2} e^{i\vec{k}\cdot\vec{r}}$$

$$\frac{d\langle P\rangle}{dt} = \frac{c}{8\pi}\left(\frac{\omega}{c}\right)^4\left(\frac{eE}{m\omega^2}\right)^2 \sin^2\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{d\langle P\rangle}{dt}\cdot\frac{1}{\frac{c}{8\pi}|E|^2} = \left(\frac{e^2}{mc^2}\right)^2 \sin^2\theta$$

$$= r_{cl}^2 \sin^2\theta$$

$$\sigma_T = \frac{8\pi}{3} r_{cl}^2 = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$$ Thompson cross section

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Lienard · Wrechart – Poleniah

results · $P(t) = \frac{2}{3}\frac{|\ddot{\vec{p}}(t)|^2}{c^3} + \frac{2}{3}\frac{|\ddot{\vec{M}}|^2}{c^3} + \frac{1}{180c^5}\sum_{\alpha\beta}|\dddot{Q}_{\alpha\beta}|^2$

exam question · Fall 02 #2

$\rightarrow \vec{B} = B_0\hat{z}$ $\quad v_0 \ll c$

$\downarrow v_0$

$$m\frac{d\vec{v}}{dt} = -e\frac{\vec{v}}{c}\times\vec{B}_0$$

$$\dot{\vec{v}} = \ddot{\vec{r}} = -\frac{e}{m}\frac{\vec{v}}{c}\times\vec{B} = -\frac{e}{m}\frac{(v B_0)}{c}$$

$$\vec{p}(t) = -e\vec{r}$$

$$\ddot{\vec{p}} = -e\ddot{\vec{r}} = \frac{e^2}{m}\frac{v B_0}{c}$$

so

$$P(t) = \frac{2}{3}\frac{e^4}{m^2}\frac{v^2 B_0}{c^5} = \frac{2}{3}\left(\frac{e^2}{mc^2}\right)^2\frac{v^2 B_0^2}{c}$$

$$KE = \frac{1}{2}mv^2$$

$$\frac{d\,KE}{dt} = -P(t)$$

$$\frac{m}{2}\,d(v^2) = -\frac{2}{3}r_{c1}^2\left(\frac{v^2 B_0^2}{c}\right)$$

$$\ln v^2\Big|_0^{t=} = -\left(\frac{4}{3}r_{c1}^2\frac{B_0^2}{mc}t\right)$$

$$\frac{m}{2}v(t)^2 = \frac{m}{2}v_0^2\exp\left(-\frac{4}{3}\cdot\frac{(r_{c1}B_0)^2}{mc}t\right)$$

Lifetime of the atom:

$$\frac{mv^2}{r} = \frac{e^2}{r^2}$$

$$E = \frac{m}{2}v^2 - \frac{e^2}{r} = \frac{e^2}{2r} - \frac{e^2}{r} = -\frac{e^2}{2r}$$

$$\dot{E} = -\frac{2}{3}\frac{e^2}{c^3}\left(\frac{v}{r}^2\right)^2$$

Solve for $r(t)$,

$$+\frac{e^2}{2r^2}\dot{r} = -\frac{2}{3}\frac{e^2}{c^3}\frac{1}{r^4}\left\{\frac{e^2}{mr}\right\}^2$$

$$\dot{r} = -\frac{4}{3}\left(\frac{e^2}{mc^2}\right)c\,\frac{1}{r^2}$$

$$\dot{r} = -\frac{4}{3}\left(\frac{r_{cl}}{r}\right)^2 c \qquad r_{cl} = \frac{e^2}{mc^2} = 2.8\times10^{-15}\ m$$

$$r^2\,dr = -\frac{4}{3}r_{cl}^2\,c\,dt$$

$$\frac{r^3}{3} - \frac{r_0^3}{3} = -\frac{4}{3}r_{cl}^2\,c\,t$$

$$r = \left(r_0^3 - 4r_{cl}^2 ct\right)^{1/3}$$

$$r(t) = 0 \quad \text{at} \quad t = \frac{r_0^3}{4cr_e^2} \approx 10^{-11}\,s$$

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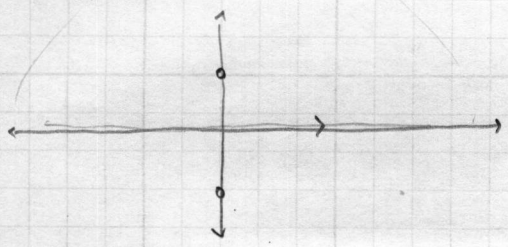
$1+u^2 = 0 \qquad u = \pm i$



$$\int_{-\infty}^{\infty} \frac{du}{(1+u^2)^2} = 2\pi i \sum_i \text{Res } f(u)$$

$$f(u) = \frac{1}{(1+u^2)^2}$$

$$\text{Res} = \frac{1}{(u+i)^2(u-i)^2}$$

pole order n: $\quad f(z) = \frac{b_n}{(z-z_0)^n} + \frac{b_{n-1}}{(z-z_0)^{n-1}} + \cdots \frac{b_1}{(z-z_0)} + a_0 + a_1 z + \cdots$

$$f(z)(z-z_0)^n = b_n + (b_{n-1}) z + \cdots b_1 (z-z_0)^{n-1} + \cdots a_0 (z-z_0)^n$$

$$\text{Res} = \left(\frac{d}{dz}\right)^{n-1} \left[(z-z_0)^n f(z) \right]$$

$$= \frac{d}{dz}\left[\frac{1}{(u+i)^2}\right]_{u=i} = \frac{-2}{(u+i)^3}\Big|_{u=i} = \frac{-2}{-8i} = -\frac{1}{4} i$$

$$\int \frac{du}{(1+u^2)^2} = 2\pi i \cdot \left(-\frac{1}{4} i\right) = \frac{\pi}{2}$$

$$\omega = \frac{\pi}{3c^3} \frac{e^6}{\mu} \frac{1}{v_0 b^3}$$

$e^- - e^-$ collisions

1. $\quad \vec{p} = -e(\vec{r}_1 + \vec{r}_2) = -\frac{e}{m}(m\vec{r}_1 + m\vec{r}_2)$

$$\dot{\vec{p}} = -\frac{e}{m}(m\vec{v}_1 + m\vec{v}_2) = \text{const}$$

$$\ddot{\vec{p}} = 0 \qquad \text{no dipole radiation}$$

2. $\quad \vec{m} = \frac{e}{2c} \vec{r}_1 \times \vec{v}_1 + \frac{e}{2c} \vec{r}_2 \times \vec{v}_2$

From $\vec{m} = \frac{1}{2c} \int \vec{r}' \times \vec{J}(\vec{r}') \, d^3\vec{r}' = \frac{e}{2c} \vec{r} \times \vec{v}$

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$$\vec{M} = \frac{e}{2mc}\left(\vec{l}_1 + \vec{l}_2\right)$$

$$\ddot{\vec{M}} = 0 \qquad \text{no magnetic radiation!}$$

lowest order:

$$W = \frac{1}{120c^5}\int dt \sum_{\alpha\beta} \left|\dddot{Q}_{\alpha\beta}\right|^2$$

Relativistic generalization

$$P = -\frac{2}{3}\left(\frac{e^2}{mc^2}\right)\frac{1}{c}\left(\frac{dp_\mu}{d\tau}\,\frac{dp^\mu}{d\tau}\right) \qquad d\tau = \frac{dt}{\gamma}$$

$$= -\frac{2}{3}\,\frac{e^2}{mc^3}\left(\frac{1}{c^2}\left(\frac{d\varepsilon}{dt}\right)^2 - \frac{d\vec{p}}{dt}\cdot\frac{d\vec{p}}{dt}\right)\gamma^2$$

$$= \frac{2}{3}\,\frac{e^2}{mc^3}\,\gamma^2\left(\left(\frac{d\vec{p}}{dt}\right)^2 - \beta^2\left(\frac{d|\vec{p}|}{dt}\right)^2\right)$$

Linear $\qquad \dfrac{d|\vec{p}|}{dt} = \dfrac{d\vec{p}}{dt}$

$$P_L = \frac{2}{3}\,\frac{e^2}{mc^3}\,\gamma^2(1-\beta^2)\left(\frac{d|\vec{p}|}{dt}\right)^2$$

$$= \frac{2}{3}\,\frac{e^2}{mc^3}\left(\frac{d|\vec{p}|}{dt}\right)^2$$

Transverse $\qquad \dfrac{d\vec{p}}{dt}\cdot\vec{p} = 0 \rightarrow$ no work

$$\rightarrow \frac{d\varepsilon}{dt} = 0$$

$$P_T = \frac{2}{3}\,\frac{e^2}{m^2c^3}\,\gamma^2\left(\frac{d\vec{p}}{dt}\right)^2$$

Note if same force used to produce linear & transverse,

$$P_T = \gamma^2 P_L$$

↑ reason for linear accelerator

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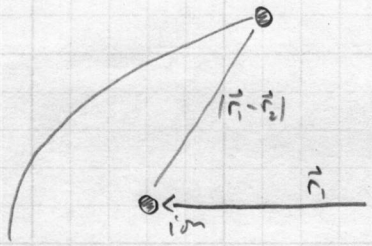
Collision problems

e^- - ion collision → Bremsstrahlung

$\vec{\rho} = e(\vec{r}_1 - \vec{r}_2) = e\vec{r}$ ← relative coordinate



$$\ddot{\vec{\rho}} = e\ddot{\vec{r}} = \frac{e\mu}{\mu}\ddot{\vec{r}} = \frac{e}{\mu}\left(-e^2\frac{\hat{r}}{r^2}\right)$$

$$= -\frac{e^3}{\mu}\frac{\hat{r}}{r^2}$$

So $$W = \int I(t)\,dt = \frac{2}{3}\frac{1}{c^3}\int_{-\infty}^{\infty} dt\ |\ddot{\vec{\rho}}|^2$$

$$W = \frac{2}{3c^3}\frac{e^6}{\mu^2}\int_{-\infty}^{\infty} dt\left[\frac{1}{r(t)}\right]^4$$

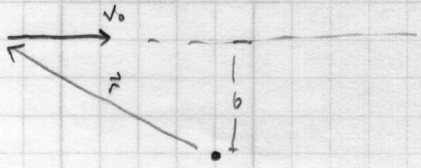
can use $\because l = \mu r^2\dot{\theta} \rightarrow d\theta = \frac{\mu r^2}{l}\,dt$, $r(\theta) = \dfrac{\left[\frac{l^2}{\mu e^2}\right]}{1 - \sqrt{1 + \frac{2El^2}{\mu e^4}}\cos\theta}$

$$W = \frac{2}{3c^3}\frac{e^6}{\mu^2}\frac{\mu}{l}\int \frac{d\theta}{r^2(\theta)}$$

exam question fall 02 #14
spring 06

$$\frac{e^2}{b} << m v_0^2 << mc^2$$

↑ impact parameter



$$|\vec{r}(t)|^2 = b^2 + (v_0 t)^2$$

$u = \frac{v_0}{b}t$

$$W = \frac{2}{3c^3}\frac{e^6}{\mu}\int_{-\infty}^{\infty}\frac{dt}{\left[b^2 + (v_0 t)^2\right]^2} = \frac{2}{3c^2}\frac{e^6}{\mu}\frac{1}{b^4}\int_{-\infty}^{\infty}\frac{dt}{\left[1 + \left(\frac{v_0}{b}t\right)^2\right]}$$

$$= \frac{2}{3c^3}\frac{e^6}{\mu}\frac{1}{b^4}\left(\frac{b}{v_0}\right)\int_{-\infty}^{\infty}\frac{du}{(1+u^2)^2}$$

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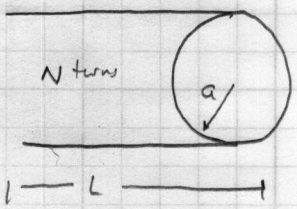
Sp 04

Solenoid



$V(t) = \mathrm{Re}\, V_0 e^{-i\omega t}$

assume $a \ll L \ll c/\omega \sim \lambda$

1. $\vec{p} = \int d^3\vec{r}\,' \; \vec{r}\,' \rho(\vec{r}\,') = 0$ (no net charge in wires)

2 find $\vec{M}$: we near field since $|\vec{r}| \sim L \ll \lambda$

$$V(t) = L \frac{\partial I}{\partial t} \qquad L = \frac{1}{c}\frac{\Phi_{tot}}{I}$$

$$= \frac{1}{c} N \frac{d\Phi_{loop}}{dt}$$

$$\mathrm{Re}\, V_0 e^{-i\omega t} = \frac{N}{c}\frac{d}{dt}\left(\pi a^2 \vec{B}\right)$$

$c = \lambda f$

$c = \frac{\omega}{k}$

$k = \frac{2\pi}{\lambda}$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\int \vec{B}\cdot d\vec{\ell} = \frac{4\pi}{c}\int \vec{J}\cdot d\vec{A} = \frac{4\pi}{c} N I$$

$$B = \frac{4\pi}{c}\frac{N I}{L}$$

$$\mathrm{Re}\left(V_0 e^{-i\omega t}\right) = \frac{N}{c} A \frac{4\pi}{c}\frac{N}{L}\left(\frac{dI}{dt}\right) = \frac{4\pi N^2 A}{c^2 L}\,\mathrm{Re}\left(-i\omega I_\omega e^{-i\omega t}\right)$$

$$I_\omega = \frac{V_0 c^2 L}{4\pi N^2 (\pi a^2)(-i\omega)}$$

$$M = \frac{N A I_\omega}{c} = \frac{V_0 c L}{4\pi N}\left(\frac{1}{-i\omega}\right)$$

fw radiation, we far field.

$$\langle P \rangle = \frac{1}{3}\frac{\omega^4}{c^3}\left|\frac{V_0 c L}{4\pi N \omega}\right|^2$$

$$= \frac{1}{48\pi^2}\frac{\omega^2 L^2 |V_0|^2}{N^2 c}$$

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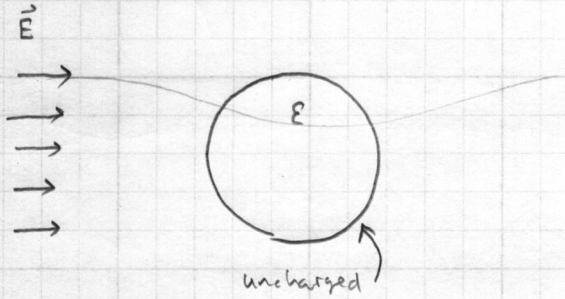
uniform $\vec{E}$ - field



1. $\varphi(a-) = \varphi(a+)$

B.C. 2. $\nabla \cdot \vec{D} = 4\pi \rho_f = 0$

$\varphi \to -E_0 r \cos\theta$ as $r \to \infty$

$\vec{D} = \varepsilon \vec{E}$

$$\varphi_{in} = \sum_\ell A_\ell r^\ell P_\ell(\cos\theta)$$

$$\varphi_{out\,side} = \tilde{A}_1 r \cos\theta + \sum_\ell \frac{\tilde{B}_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

$$\nabla \cdot D = 0 \to \quad \varepsilon \left.\frac{\partial \varphi_{in}}{\partial r}\right|_a = \left.\frac{\partial \varphi_{out}}{\partial r}\right|_a$$

1. $\varphi(a-) = \varphi(a+)$

$$A_1 a \cos\theta + \sum_{\ell>2} A_\ell a^\ell P_\ell(\cos\theta) = \left(\tilde{A}_1 a + \frac{\tilde{B}_1}{a^2}\right)\cos\theta + \sum_\ell \frac{\tilde{B}_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

$$A_1 a = \tilde{A}_1 a + \frac{\tilde{B}_1}{a^2}$$

$$A_\ell a^\ell = \frac{\tilde{B}_\ell}{a^{\ell+1}} \to A_\ell = \frac{\tilde{B}_\ell}{a^{2\ell+1}}$$

2. $$\left(\tilde{A}_1 - 2\frac{\tilde{B}_1}{a^3}\right)\cos\theta - \sum (\ell+1)\frac{\tilde{B}_\ell}{a^{\ell+2}} P_\ell(\cos\theta) = \varepsilon A_1 \cos\theta + \sum_{\ell>2} \ell A_\ell a^{\ell-1} P_\ell(\cos\theta)$$

$$\tilde{A}_1 - 2\frac{\tilde{B}_1}{a^3} = \varepsilon A_1$$

$$\ell A_\ell a^{\ell-1} = (\ell+1)\frac{\tilde{B}_\ell}{a^{\ell+2}}$$

$$A_\ell = \left(\frac{\ell+1}{\ell}\right)\frac{\tilde{B}_\ell}{a^{2\ell+1}} \to \tilde{B}_\ell = A_\ell = 0 \quad \ell > 2$$

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$$\tilde{A}_1 = -E_0$$

So $$-\varepsilon A_1 + \frac{\tilde{B}_1}{a^3} = -\varepsilon E_0$$

$$\varepsilon A_1 + 2\frac{\tilde{B}_1}{a^3} = -E_0$$

$$\rightarrow \quad (2+\varepsilon) A_1 = -3E_0$$

$$A_1 = -\frac{3E_0}{(2+\varepsilon)}$$

$$\tilde{B}_1 (\varepsilon + 2) = (\varepsilon - 1) E_0$$

$$\tilde{B}_1 = \left(\frac{\varepsilon - 1}{\varepsilon + 2}\right) E_0$$

$$\varphi_{in} = -\frac{3E_0}{(2+\varepsilon)} r\cos\theta \quad \rightarrow \quad \vec{E}_{in} = \frac{3E_0}{(2+\varepsilon)}\hat{z} = \left(\frac{3}{2+\varepsilon}\right)\vec{E}_0$$

$$\varphi_{out} = -E_0 r\cos\theta + \left(\frac{\varepsilon-1}{\varepsilon+2}\right)\frac{E_0}{r^2}\cos\theta$$

$$\rightarrow \quad \vec{E}_{out} = E_0\hat{z} + E_0\left(\frac{\varepsilon-1}{\varepsilon+2}\right)\left(-2\frac{\cos\theta}{r^3}\hat{r} - \frac{E_0}{r^3}\sin\theta\,\hat{\theta}\right)$$

$$= E_0\hat{z} + E_0\left(\frac{\varepsilon-1}{\varepsilon+2}\right)\left(\frac{-3\cos\theta\,\hat{r} + E_0\hat{z}}{r^3}\right)$$

↑
dipole

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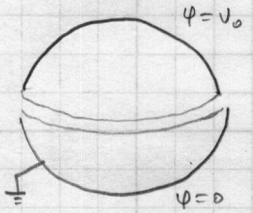

find φ inside,

$$\varphi = \sum_{l=0}^{\infty} \sum_{l=-m}^{m} \left(A_{lm} r^l + \frac{B_{lm}}{r^{l+1}} \right) Y_{lm}(\theta, \varphi)$$

$\frac{B_{lm}}{r^{l+1}} \to 0$

$$\varphi(a, \theta, \varphi) = \sum_{lm} A_{lm} a^l Y_{lm}(\theta, \varphi)$$

$$A_{lm} = \frac{1}{a^l} \int d\Omega \; \Phi(a, \theta, \varphi)$$

$$= \frac{2\pi}{a^l} \int_0^1 du \cdot \sqrt{\frac{2l+1}{4\pi}} \, P_l(u)$$

$$= \frac{2\pi}{a^l} \sqrt{\frac{2l+1}{4\pi}} \int_0^1 P_l(u) \, du$$

$$= \begin{cases} \dfrac{2\pi V_0}{a^l} \sqrt{\dfrac{2l+1}{4\pi}} \left(-\frac{1}{2}\right)^{\frac{l-1}{2}} \dfrac{(l-2)!!}{2\left(\frac{l+1}{2}\right)!} & l \text{ odd} \\ 0 & l \text{ even} \end{cases}$$

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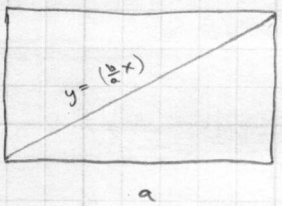

$$E_z = \sum a_{mn} \sin\left(n\frac{\pi}{a}x\right) \sin\left(n\frac{\pi}{b}y\right)$$

$$E_z\left(x, \frac{a}{b}x\right) = 0 = \sum_{mn} a_{mn} \sin\left(n\frac{\pi}{a}x\right) \sin\left(m\frac{\pi}{a}x\right) = 0$$

$$= \frac{1}{2}\left(\sum_{mn} a_{mn} + \sum_{mn} a_{nm}\right) \sin\left(m\frac{\pi}{a}x\right)\sin\left(m\frac{\pi}{a}x\right)$$

$$a_{mn} + a_{nm} = 0$$

$$a_{mn} = -a_{nm}$$

$$E_z(x,y) = \frac{1}{2}\sum_{mn} a_{mn} \sin\left(n\frac{\pi}{a}x\right) \sin\left(m\frac{\pi}{b}y\right) + \frac{1}{2}\sum a_{nm} \sin\left(m\frac{\pi}{a}x\right) \sin\left(n\frac{\pi}{b}y\right)$$

$$= \sum \frac{a_{mn}}{2} \left\{\sin\left(n\frac{\pi}{a}x\right)\sin\left(m\frac{\pi}{b}y\right) - \sin\left(m\frac{\pi}{a}x\right)\sin\left(n\frac{\pi}{b}y\right)\right\}$$

if $n = m \rightarrow a_{mn} = 0$

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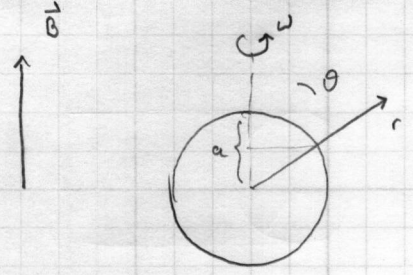
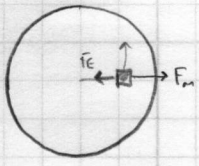


uncharged conductor, find steady state $\vec{E}$, ρ, σ

in frame of charge, no force

top view



$$\left(-M\frac{v^2}{r}\right)\hat{r} = m_e r_\perp \omega^2 = q\left(E - \frac{vB}{c}\right)(-\hat{r})$$

$$E = \frac{\omega B}{c} r_\perp + \frac{m_e}{q} r_\perp \omega^2$$

if $\frac{m_e \omega}{q} << \frac{B}{c}$

$\omega << \frac{qB}{m_e c} = \omega_c$, neglect inertia term

so $\vec{E} = \frac{\omega B}{c} r_\perp (-\hat{r})$

$\varphi = \frac{\omega B}{c} \frac{r_\perp^2}{2} + C$ $\quad \int r^2$

$$-4\pi\rho = \nabla^2\varphi = \frac{1}{r}\frac{d^2}{dr^2}\left(\frac{\omega B}{c}\frac{r^3}{2}\sin^2\theta\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\frac{\omega B}{2c} r^2\sin^2\theta\right)$$

$$= 6\frac{\omega B}{2c}\sin^2\theta + \frac{\omega B}{2c}\left[\frac{1}{\sin\theta}\frac{2}{\partial\theta}\left(2\sin^2\theta\cos\theta\right)\right.$$

$$= \frac{\omega B}{c}\left(3\sin^2\theta + \frac{1}{\sin\theta}\left(2\sin\theta\cos^2\theta - \sin^3\theta\right)\right)$$

$$= 2\frac{\omega B}{c}$$

$$\rho = -\frac{\omega B}{2\pi c}$$

$$\varphi_{in} = \frac{\omega B}{2c} r^2 (1 - \cos^2\theta) + C$$

$$\Phi_{out} = \sum_\ell \left(\cancel{A_\ell r^\ell} + \frac{B_\ell}{r^{\ell+1}} \right) P_\ell(\cos\theta)$$

$A_\ell \to 0$

$\varphi_{out} \to 0$ as $r \to \infty$,

1. $\phi(a^-, \theta) = \phi(a^+, \theta)$

2. $$0 = 4\pi Q = \int\int dS \left(-\frac{\partial \Phi_{out}}{\partial r} \right)$$

$$= \int d\Omega \, R^2 \left(\frac{B_0}{R} + \underbrace{\frac{B_1 P_1(\cos\theta)}{R^2} + \cdots}_{\text{these terms killed by } \int d\Omega} \right)$$

$$= 4\pi R B_0$$

So $B_0 = 0$

$$C + \frac{\omega B}{2c} a^2 (1 - \cos^2\theta) = \frac{B_1}{a^2} \cos\theta + \frac{B_2}{a^3} \frac{(3\cos^2\theta - 1)}{2} + \cdots$$

$B_1 = 0$

$$\frac{3B_2}{2a^3} = -\frac{\omega B}{2c} a^2 \rightarrow B_2 = -\frac{\omega B}{3c} a^5$$

$$C + \frac{\omega B a^2}{2c} = -\frac{B_2}{2a^3} = \frac{\omega B a^2}{6c}$$

$$C = \frac{\omega B a^2}{c} \left(\frac{1}{6} - \frac{3}{6} \right) = -\frac{\omega B a^2}{3c} = \frac{B_2}{a^2}$$

$$\varphi = \begin{cases} -\frac{\omega B a^2}{3c} + \frac{\omega B}{2c} r^2 (1 - \cos^2\theta) & r < a \\ -\frac{\omega B}{3c} \cdot \frac{a^5}{r^3} P_2(\cos\theta) & r > a \end{cases}$$

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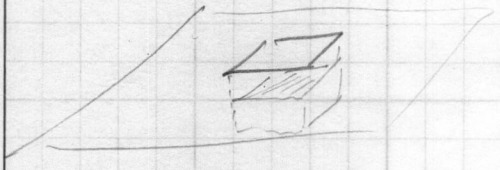
$$\left.\frac{\partial \Psi_{in}}{\partial r}\right|_a = \frac{\omega B}{c} a (1 - \cos^2\theta) = \frac{\omega B}{c} a \left(-\frac{2}{3} P_2 + \frac{2}{3} \right)$$

$$\left.\frac{\partial \Psi_{out}}{\partial r}\right|_a = -\frac{\omega B}{3c} \cdot 3 r^{-4} a^5 \left(\frac{3\cos^2\theta - 1}{2} \right)$$

$$= \frac{\omega B a}{2c} \left(3\cos^2\theta - 1 \right)$$



$$4\pi\sigma = \hat{n} \cdot \vec{E}_{top} + (-\hat{n}) \cdot \vec{E}_{bottom}$$

$$= -\left.\frac{\partial \Psi}{\partial r}\right|_{a^+} + \left.\frac{\partial \Psi}{\partial r}\right|_{a^-}$$

$$\frac{\partial \Psi_{in}}{\partial r} - \frac{\partial \Psi_{out}}{\partial r} = \frac{\omega B a}{c} \left(-\frac{2}{3} P_2 + \frac{2}{3} + P_2 \right\}_2 \Big)$$

$$\sigma = \frac{1}{4\pi} \left[\frac{\omega B a}{c} \left(\frac{2}{3} - \frac{5}{2} P_2 \right) \right]$$

$$\int P_2 \, d\Omega = 2\pi \int_{-1}^{1} du \left(\frac{3u^2 - 1}{2} \right)$$

$$= 2\pi \left[\frac{3}{2} \cdot \frac{2}{3} - \frac{1}{2}(2) \right] = 0$$

$$Q = \int ds = \frac{4\pi a^2}{4\pi} \cdot \frac{2}{3} \frac{\omega B c}{c} a$$

$$= \frac{2}{3} \frac{\omega B c}{c} a^3$$

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$$\nabla \times \vec{E} = i\omega \frac{\vec{B}}{c} \Rightarrow \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = ik E_y - \frac{\partial E_z}{\partial z} = \frac{i\omega}{c} B_x$$

$$ik E_x - \frac{\partial E_z}{\partial z} = \frac{i\omega}{c} B_y$$

and

$$\nabla \times \vec{B} = -\frac{i\omega}{c} \vec{E} \qquad ik B_y = -\frac{i\omega}{c} E_x$$

$$-ik B_x = -\frac{i\omega}{c} E_y$$

so $$E_x = \frac{c}{\omega k} B_y = \frac{c}{\omega k} \left(\frac{i\omega}{c} E_x + \frac{\partial E_z}{\partial y} \right)$$

$$E_x = ik \frac{\partial E_z}{\partial x} \left(\frac{1}{k^2 - \left(\frac{\omega}{c}\right)^2} \right)$$

similarly $$E_y = -ik \frac{\partial E_z}{\partial y} \left(\frac{1}{k^2 - \left(\frac{\omega}{c}\right)^2} \right)$$

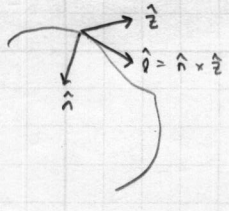
$$B_y = -\frac{i\omega}{c} \frac{\partial E_z}{\partial x} \left(\frac{1}{k^2 - \left(\frac{\omega}{c}\right)^2} \right)$$

$$B_x = -\frac{i\omega}{c} \frac{\partial E_z}{\partial y} \left(\frac{1}{k^2 - \left(\frac{\omega}{c}\right)^2} \right)$$

only need to solve E_z: $\left[\nabla_\perp + \left(\frac{\omega}{c}\right)^2 - k^2 \right] E_z(x,y) = 0$

need B.C. (1) $0 = \hat{n} \cdot \vec{B}$

$$\propto \left(\frac{i\omega}{c}\right) \frac{1}{\left[k^2 - \left(\frac{\omega}{c}\right)^2\right]} \left\{ -v_x \frac{\partial E_z}{\partial y} + v_y \frac{\partial E_z}{\partial x} \right\}$$



$$\propto \hat{z} \cdot \left\{ -\hat{n} \times \nabla_\perp E_z \right\}$$

$$= (\hat{z} \times \hat{n}) \cdot \nabla_\perp E_z$$

$$= \hat{\theta} \cdot \nabla_\perp E_z$$

$$= \frac{\partial E_z}{\partial \theta} = 0 \rightarrow E_z = \text{const on boundary},$$

but E_z is tangential $\rightarrow$ $E_z = 0$

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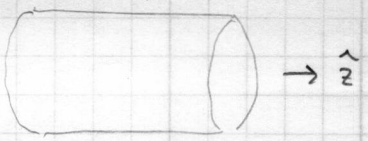
Cylindrical wave guide



TM: $\left[\nabla_\perp + \left(\frac{\omega}{c}\right)^2 - k^2\right] E_z = 0$

$$\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \lambda^2\right\} E_z = 0$$

$$E_z \sim E(r)\left(A_m \cos m\theta + B_m \sin m\theta\right)$$

$$\frac{\partial^2 E_z}{\partial\theta^2} = -m^2 E_z$$

$$\left\{r\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) - m^2 + r^2\lambda^2\right\} E(r) = 0 \quad \Rightarrow \quad E(r) = J_m(\lambda r)$$

$x = r\lambda \qquad \frac{\partial}{\partial r} = \frac{\partial}{\partial x}\frac{\partial x}{\partial r} = \lambda\frac{\partial}{\partial x}$

$$r\frac{\partial}{\partial r} = x\frac{\partial}{\partial x}$$

$$\left[x\frac{\partial}{\partial x}\left(x\frac{\partial}{\partial x}\right) - m^2 + x^2\right] E(r) = 0$$ Bessels equation

$$E_z = \sum_m J_m(\lambda r)\left(A_m \cos m\theta + B_m \sin m\theta\right)$$

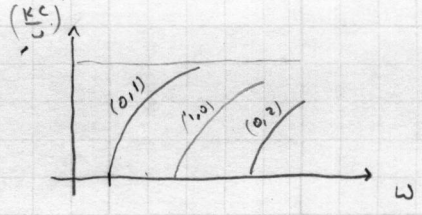
$E_z(r=a) = 0 \rightarrow \quad J_m(\lambda a) = 0$

$\lambda a = \gamma_{mn}$ (n^{th} zero of order m bessel func.)

$\lambda = \frac{\gamma_{mn}}{a}$

So $\left(\frac{\omega}{c}\right)^2 - k^2 = \left(\frac{\gamma_{mn}}{a}\right)^2$ or $\frac{kc}{\omega} = \sqrt{1 - \left(\frac{c\gamma_{mn}}{\omega a}\right)^2}$

$$\omega = c\sqrt{k^2 + \left(\frac{\gamma_{mn}}{a}\right)^2}$$



$\gamma_{01} = 2.405$

$\gamma_{10} = 3.832$

$\gamma_{02} = 5.520$

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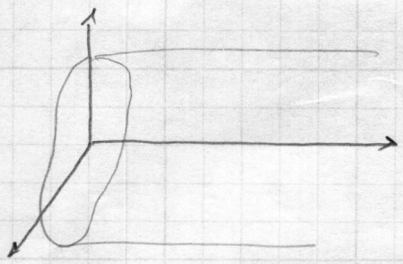
Wave guides



B. C. for perfect conductor

$$\hat{n}\cdot\vec{B}=0 \qquad \hat{n}\times\vec{E}=0$$

$$\hat{n}\cdot\vec{E}=4\pi\sigma \qquad \hat{n}\times\vec{B}=\frac{4\pi}{c}\vec{K}$$

assume $\vec{E}$ & $\vec{B}$ harmonic! $\vec{B},\vec{E}\sim e^{-i\omega t}$

$$\nabla\times\vec{E}=\frac{i\omega}{c}\vec{B}$$

$$\nabla\times\vec{B}=-\frac{i\omega}{c}\vec{E}$$

$$\nabla\cdot\vec{E}=0$$

$$\nabla\cdot\vec{B}=0$$

so $\left(\nabla^2-\frac{1}{c}\frac{\partial^2}{\partial t}\right)\rightarrow\left(\nabla^2+\frac{\omega^2}{c^2}\right)$,

assume translational invariance in z, $\vec{B},\vec{E}\sim e^{+ikz}$

$$\nabla^2=\partial_x^2+\partial_y^2-k^2\equiv\nabla_\perp^2-k^2$$

so solve $\left\{\nabla_\perp^2-k^2+\frac{\omega^2}{c^2}\right\}\vec{E}\text{ or }\vec{B}=0$

a) TM mode: $\vec{B}_z=0$ (non-zero $\vec{B}$ in transverse direction only)

b) TE mode: $\vec{E}_z=0$ (non-zero $\vec{E}$ in transverse direction only)

c) TEM mode: both

use M.E. to find all components

$$\left[\nabla_\perp + \lambda^2\right] E_z = 0 \qquad \text{w/} \quad E_z = 0 \text{ on Boundary}$$

Dirchlét B.C.

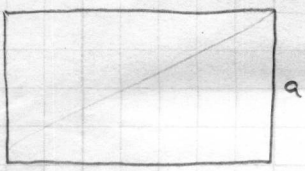
TE mode $\quad \vec{E}_z = 0$ everywhere

$$\left[\nabla_\perp + \left(\frac{\omega}{c}\right)^2 - K^2\right] B_z = 0 \qquad \hat{n}\cdot\nabla\vec{B} = 0 \text{ on boundary}$$

$$B_x = \frac{iK\,\partial_x B_z}{\left(\frac{\omega}{c}\right)^2 - K^2} \qquad B_y = \frac{iK\,\partial_y B_z}{\left(\frac{\omega}{c}\right)^2 - K^2}$$

$$E_x = \frac{i\omega}{c}\frac{\partial_y B_z}{\left(\frac{\omega}{c}\right)^2 - K^2} \qquad E_y = -\frac{i\omega}{c}\frac{\partial_x B_z}{\left(\frac{\omega}{c}\right)^2 - K^2}$$

fall 04 # 14



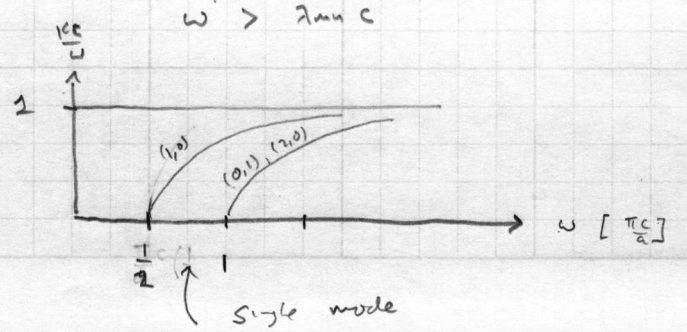
a) TM

$$E_z = E_0 \sin\left(n\frac{\pi}{2a}x\right)\sin\left(m\frac{\pi}{a}y\right)$$

$$\lambda_{mn}^2 = n^2\left(\frac{\pi}{2a}\right)^2 + m^2\left(\frac{\pi}{a}\right)^2$$

$$\lambda_{mn}^2 = \left(\frac{\pi}{a}\right)^2\left(\left(\frac{n}{2}\right)^2 + m^2\right) = \left(\frac{\omega}{c}\right)^2 - K^2$$

$$K = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2\left(\left(\frac{n}{2}\right)^2 + m^2\right)}$$

$$\frac{Kc}{\omega} = \sqrt{1 - \left(\frac{\lambda_{mn}c}{\omega}\right)^2} > 0$$

$$\omega > \lambda_{mn}c$$