

SCORE _____

Physics Departmental Examination - Fall 1995 PART I

Student Identification # _____

- [1] Consider a system of 2 springs and 3 masses as shown below



Calculate the eigenfrequencies of the system. Show what happens for $m/M \rightarrow \infty$.

Physics Departmental Examination - Fall 1995 PART I

Student Identification # _____

[2] Quick Estimate Problems

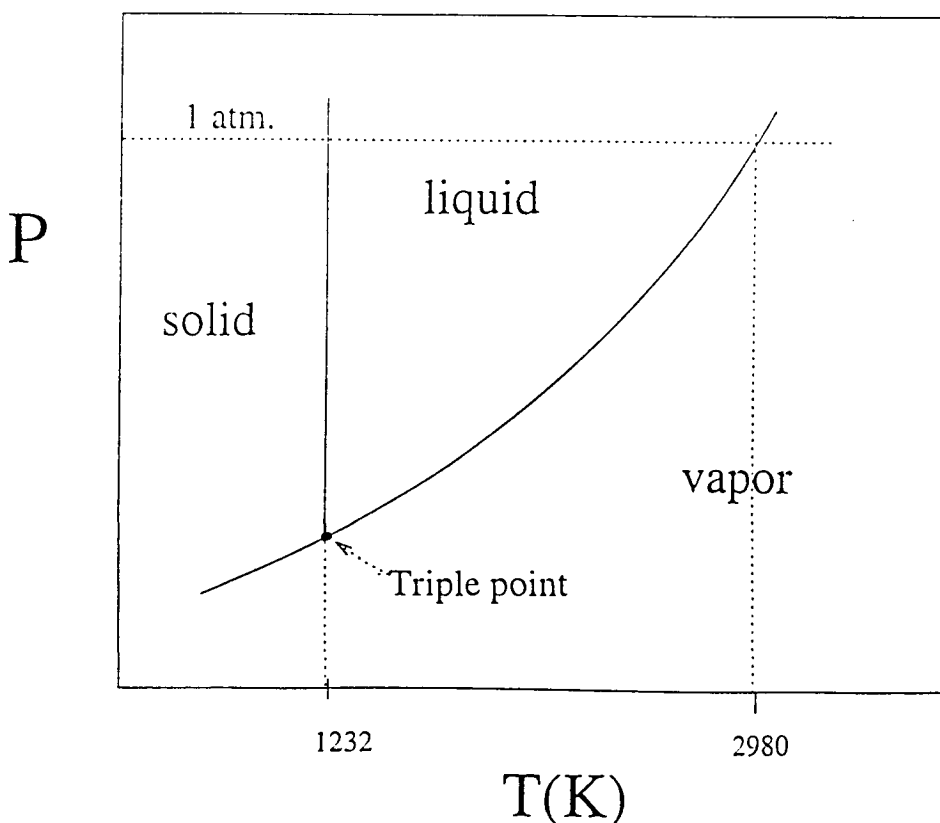
- (a). What is the De Broglie wavelength of a proton that has been accelerated through a potential difference of 1 GeV.
- (b) Calculate the ionization potential of a positronium, a bound state of an electron and positron.
- (c) Monochromatic light is incident upon a perfectly reflecting surface with an incident angle α . The energy density of the light is w . Calculate the light pressure on the surface.
- (d) In a study of the hyperfine structure of an atom, a 3P_1 to 1S_0 transition is studied. It is found that the emitted light consists of a triplet of closely spaced lines. What can you say about the spin of the nucleus?

Physics Departmental Examination - Fall 1995 PART I

Student Identification # _____

[3] (a) Consider a first-order phase boundary in the pressure versus temperature plane. Deduce from first principles the slope at a point on the phase boundary in terms of discontinuities of relevant thermodynamic properties across the phase transition. (The relation is known as the Clausius-Clapeyron equation.)

(b) Consider the phase diagram of pure Ge as seen in the figure below. At 1 atm. pressure, the melting point of pure Ge is 1232 K and the boiling point is 2980 K. The pressure at the triple point of solid, liquid and vapor phases is 8.4×10^{-8} atm. Using the Clausius-Clapeyron equation, estimate the heat of vaporization of Ge. You may assume that (i) the volume change on melting is negligible, (ii) the specific volume of the liquid is much smaller than that of the vapor, which may be treated as an ideal gas, and (iii) the heat of vaporization to be independent of temperature and pressure. [The universal gas constant is $R = 8.31 \text{ J/mol-K}$.] Make additional approximations if needed.



Physics Departmental Examination - Fall 1995 PART I

Student Identification # _____

- [4] A conducting sphere of radius R carries charge Q . If the sphere were cut in half, what force would be required to hold the two halves together?

Hint: You may wish to consider the stress

$$T_{ij} = \frac{1}{4\pi} \left[E_i E_j - \frac{1}{2} \delta_{ij} \bar{E}^2 \right]$$

across the plane that bisects the sphere.

Physics Departmental Examination - Fall 1995 PART I

Student Identification # _____

- [5] Consider an electromagnetic plane wave normally incident on a planar metallic surface. Suppose that the metal is a good conductor, i.e. $\frac{4\pi\sigma}{\omega\epsilon} \gg 1$ where σ is the conductivity.

- (a) Show that inside the metal

$$\hat{E}(\vec{r}, t) = \vec{E}_0 \exp\{+(1-i)\hat{n}\cdot\vec{r} / \delta - i\omega t\}$$

where \hat{n} is the outward surface normal.

- (b) Derive an explicit expression for δ in terms of σ , μ , ϵ and ω . Assume that σ , μ , and ϵ are frequency and space independent.

Physics Departmental Examination - Fall 1995 PART I

Student Identification # _____

- [6] Consider a spin 1/2 object in the presence of a constant magnetic field $\vec{B}_1 = B_0 \hat{z}$. At $t = 0$, an additional time-dependent field is turned on $\vec{B} = A \cos \omega t \hat{x}$, and this perturbation lasts for time T . Assume that the system starts at $t = 0$ in the ground state $|\downarrow\rangle$ of the Hamiltonian $H = -\mu_0 \vec{S} \cdot \vec{B}$.
- a) Find the resonant value of ω , i.e. that value which gives rise to a maximal occupation of the excited state at time t , assuming A is small.
- b) Write down (but do not solve!) the coupled equations governing the time-evolution of the coefficients a and b , where

$$|\psi(t)\rangle \equiv a|\downarrow\rangle + b|\uparrow\rangle$$

Physics Departmental Examination - Fall 1995 PART I

Student Identification # _____

- [7]. a) A hydrogen atom is in a uniform electric field in the z direction which turns on abruptly at $t = 0$ and decays exponentially as a function of time, $E(t) = E_0 e^{-t/\tau}$. The atom is initially in its ground state. Find the probability for the atom to have made a transition to the 2P state at $t \rightarrow \infty$. What z components of orbital angular momentum are allowed in the 2P states generated by this transition? You may find the following radial wave functions useful.

$$R_{10} = 2\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0} \quad R_{20} = 2\left(\frac{Z}{2a_0}\right)^{3/2} \left(1 - \frac{Zr}{2a_0}\right) e^{-Zr/2a_0} \quad R_{21} = \frac{4}{\sqrt{3}} \left(\frac{Z}{2a_0}\right)^{3/2} \left(\frac{Zr}{a_0}\right) e^{-Zr/2a_0}$$

In addition, you may wish to use $\langle \psi_{210} | z | \psi_{000} \rangle = K_0 a_0$

$$\text{where } K_0 \equiv \frac{2^7 \sqrt{2}}{35}$$

- b) A hydrogen atom is in the ${}^5D_{5/2}$ state. To which states is it allowed to decay via electric dipole transitions? What will be the polarization for a photon emitted along the z -axis if m_l decreases by one unit in the decay?

Physics Departmental Examination - Fall 1995 PART I

Student Identification # _____

- [8] A pathetically over-simplified model of a white dwarf star might be that of a non-relativistic degenerate Fermi gas bound by gravitational attraction.
- a) Calculate the chemical potential of the degenerate fermi gas of electrons with density ρ at $T = 0$.

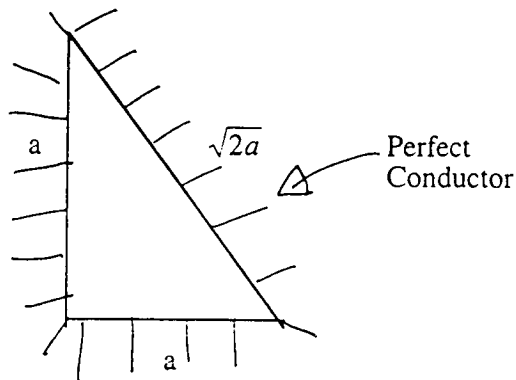
 - b) Use part (a) to derive a differential equation for the force of gravity on a test mass at all points within the white dwarf.

Physics Departmental Examination - Fall 1995 PART II

Student Identification # _____

- [9] What mode will propagate at the lowest possible frequency, the TE or TM mode, for a waveguide constructed in the form of an isosceles right triangle (see figure).

HINT: Write the solution for both situations.



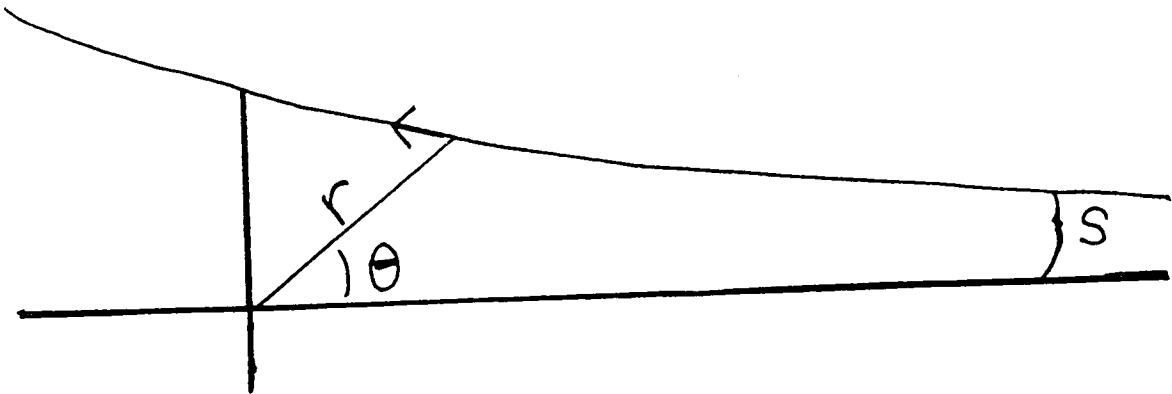
Physics Departmental Examination - Fall 1995 PART II

Student Identification # _____

- [10] A particle moves in a plane under the action of the potential

$$V(r, \theta) = \frac{a}{r} + \frac{b \sin \theta}{r^2}$$

where (r, θ) are polar coordinates and a and b are constants. The particle moves along the trajectory shown below, and is incident (at $r = \infty$) with parameter s and velocity v_0 . Because the potential depends on θ , the conjugate momentum p_θ is not constant of the motion. Determine p_θ when the particle has moved along the trajectory from $\theta = 0$ to $\theta = \pi/2$.



Physics Departmental Examination - Fall 1995 PART II

Student Identification # _____

- [11] An ideal gas of N spin 0 particles is confined to move in one dimension in an external potential. The potential is such that the energy eigenvalues of a single particle are

$$E = \epsilon n^\alpha, \quad n = 0, 1, 2, \dots$$

where α is a positive real number and ϵ is a constant energy.

- a) Write the general expression for conservation of particles using the continuum approximation for the energy levels.
- b) Find a range of α values for which a Bose-Einstein phase transition occurs in this system, and find an expression for the critical temperature T_c . You may find the following integral useful:

$$\int_0^\infty \frac{x^{\nu-1} dx}{e^x - 1} = \Gamma(\nu) \zeta(\nu)$$

where $\zeta(\nu)$ is the Riemann zeta function.

Physics Departmental Examination - Fall 1995 PART II

Student Identification # _____

- [12] Consider an ensemble of damped harmonic oscillators, each of which is governed by the equation of motion $m\ddot{x} + \lambda\dot{x} + kx = 0$. The state of any oscillator is specified by a point in the phase space (x,p) , where $p = m\dot{x}$. Let $D(x,p,t)$ be the phase space density distribution for the ensemble (i.e., $D = dN/dA$, where $dA = dx dp$).
- a) Obtain a partial differential equation which governs the temporal evolution of $D(x,p,t)$. Note that this equation is a modified version of Liouville's equation, the modification occurring because damped oscillators are not Hamiltonian.
- b) Solve the equation to find the time evolution of $D(x,p,t)$ along a trajectory. What does this imply about the time evolution of area $[R(t)]$, where $R(t)$ is some region of phase space which evolves under the dynamics?

SCORE _____

Physics Departmental Examination - Fall 1995 PART II

Student Identification # _____

[13] Consider the differential equation

$$\frac{d^2y}{dx^2} = x^2y$$

a) Expanding around $x = 0$, express the two solutions as infinite series expansions,

i.e.. $y = \sum_{n=0}^{\infty} a_n x^{\alpha n + \beta}$ Determine α, β , and the a_n 's.

b) Find the 2 possible asymptotic behaviors of y at ∞ .

c) If $y \rightarrow 0$, as $x \rightarrow \infty$, determine $y'(0)/y(0)$

Physics Departmental Examination - Fall 1995 PART II

Student Identification # _____

- [15] Calculate the differential cross section, $\frac{d\sigma}{d\Omega}$, for high energy scattering of particles of mass m and momentum p , from a spherical shell delta function

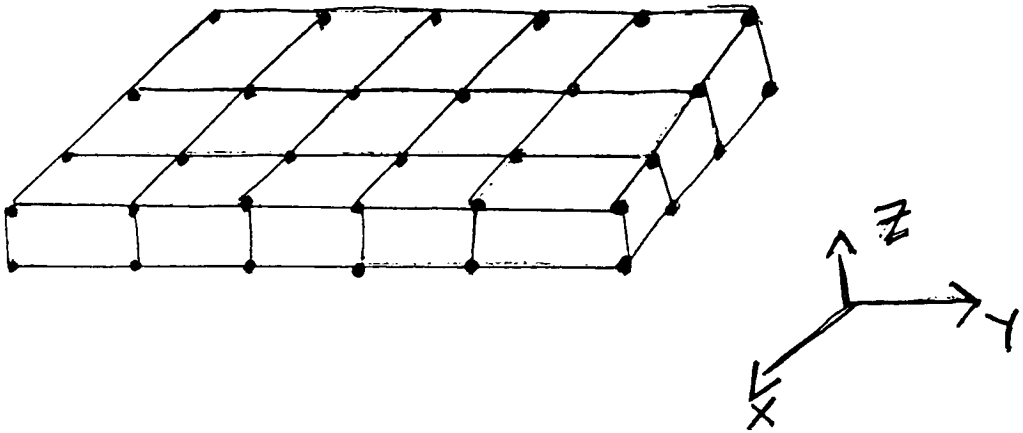
$$V(R) = \lambda\delta(r - r_0)$$

Assume that the potential is weak so that perturbation theory can be used. Be sure to write your answer in terms of the scattering angles. Crudely plot the differential cross section at 90° scattering angle as a function of p .

Physics Departmental Examination - Fall 1995 PART II

Student Identification # _____

- [16] Consider a particle of mass M scattering off a very large rectangular lattice with lattice spacing d in the x and y directions



and lattice spacing D in the z direction.

Suppose the particle interaction with the lattice points is modeled by a potential

$$V = - \frac{2\pi A \hbar^2}{M} \sum_i \delta(\vec{r} - \vec{r}_i) \quad \text{where } A \text{ is a constant}$$

and the sum is over all lattice points.

Using the first order Born approximation, find the angles at which non-vanishing scattering occurs.