DEPARTMENT OF PHYSICS
UNIVERSITY OF CALIFORNIA AT SAN DIEGO
LA JOLLA, CALIFORNIA 92093-0354

WRITTEN DEPARTMENTAL EXAMINATION, FALL 1992

PART I

Identification Number: 59

Instructions

Each problem is worth 10 points. Part I has 8 problems (numbers 1 through 8).

4

UCSD Physics Department Written Exam

Fall 1992 – Part I

Identification Number: 59

Problem 1

In this problem, you are asked to give numerical answers. If there are quantities whose values you don't know, leave the answers as formulae, clearly defining any unknown constants.

1 (a) Estimate the speed of a neutron in an atomic nucleus. Make clear your assumptions.

3 (b) A closed container at room temperature holds liquid water in equilibrium with its vapor (saturated vapor pressure ≈ 0.026 atm; 1 atm $\approx 10^6\,\mathrm{dyn/cm^2}$). Make a crude estimate (*i.e.*, to within a factor of 3) of the time necessary for the number of molecules in a monolayer of liquid at the surface to exchange with an equal number of molecules in the vapor. You may assume that every incident water molecule from the vapor sticks to the liquid surface.

10

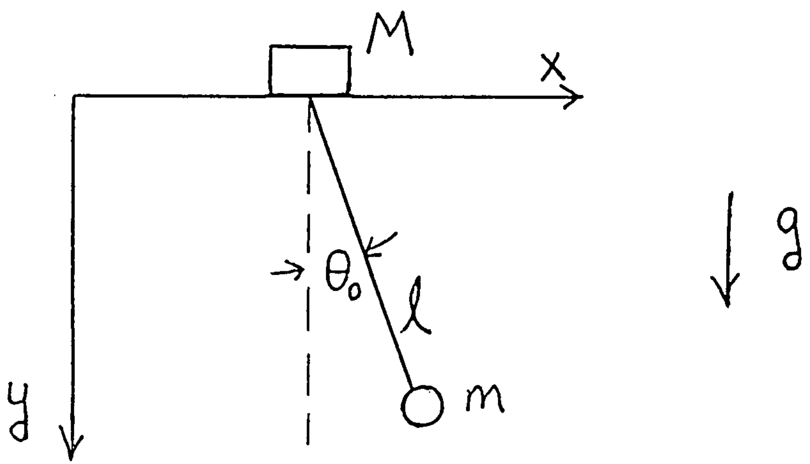
UCSD Physics Department Written Exam

Fall 1992 - Part 1

Identification Number: 59

Problem 2

A pendulum of length ℓ and mass m is suspended from a block of mass M. The block moves without friction and is constrained to move in the horizontal direction only (ie., along the x axis). You may assume all motion is confined to the xy plane. At $t = 0$, both masses are at rest, the horizontal coordinate of the block is x_0, and the pendulum has angular deflection $\theta = \theta_0$ with respect to the y axis as shown. Assuming $\theta_0 \ll 1$, calculate the subsequent motion of the block and pendulum.



8

UCSD Physics Department Written Exam

Fall 1992 - Part I

Identification Number: 59

Problem 4

Five easy pieces. Each answer must be supported by brief reasoning.

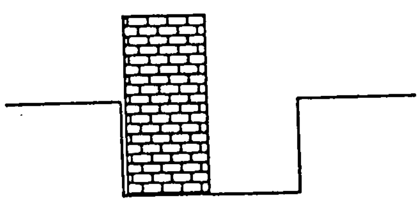
(a) A positron is an elementary particle just like an electron but with positive charge. Positronium is a neutral bound state consisting of one positron and one electron. By analogy with the hydrogen atom, find the ground state binding energy of positronium. Give your answer in electron Volts.

(b) Consider a particle in the presence of a spherically symmetric potential, $V(r)$. One of the eigenstates of energy E corresponds to the eigenfunction $z\phi(r)$. Are the states (i) $x\phi(r)$ and (ii) $r\phi(r)$ necessarily eigenstates with the same energy E?

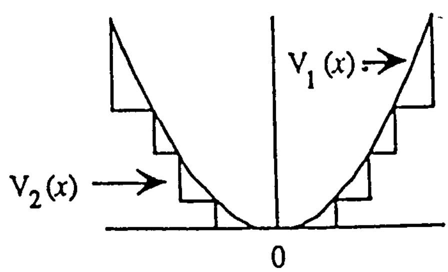
(c) Tunneling currents are measured under identical conditions in three tunnel diodes which have barriers of varying thickness d but are otherwise made of identical materials. The table below gives the tunnel currents in two of the samples. Find the tunnel current in sample C:

Sample	d(Å)	Current (μA)
A	10	10
B	20	1
C	30	?

(d) A one-dimensional square well has one and only one bound state. If an impenetrable barrier is introduced to block off half the well, how many bound states are there in the new well?



(e) In the figure shown below, $V_1(x)$ is a quadratic potential and $V_2(x)$ a series of steps. Which has a lower ground state energy? Why?



$\to \frac{1}{2}m$

$2T = V$

$2T$

T

UCSD Physics Department Written Exam

Fall 1992 – Part I

Excellent

Identification Number: 59

Problem 5

Two neutral particles each have spin $S = \frac{1}{2}$. They interact through a Hamiltonian of the form $H = -J\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$, where $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ are the vector spin operators for the two particles and H is a positive constant.

(a) Find the energy levels and eigenfunctions of the system, assuming the particles to be distinguishable.

(b) A static external magnetic field B is now introduced. Assuming the particles have the same gyromagnetic ratio, find the energy levels of the system in the presence of the field, again assuming the particles to be distinguishable.

(c) Now suppose that the two particles are identical. Find the ground state energy and wave function for the system in the absence of a magnetic field.

10

UCSD Physics Department Written Exam

Fall 1992 – Part I

Identification Number: 59

Problem 6

A long rectangular bar at temperature T exerts a tension $\tau(\xi, T)$ when extended by a distance ξ beyond its natural length.

(a) What is the thermodynamic relation between dS, T, dE, $\tau(\xi, T)$, and $d\xi$, where S is the entropy and E the internal energy of the bar?

(b) What is the relationship between $(\partial S/\partial \xi)_T$ and $(\partial \tau/\partial T)_\xi$ for a quasistatic process?

(c) Let $\tau(\xi, T) = b\xi(1 - \gamma T)$, where b and γ are positive constants. What is the change in internal energy of the bar when it is stretched from $\xi = 0$ to $\xi = \xi_0$ at constant temperature?

(d) What are the sign and magnitude of the heat, Q, required to maintain the bar at constant temperature for the process described in (c)?

UCSD Physics Department Written Exam

Fall 1992 - Part I

Identification Number: 59

Problem 7

A hydrogen atom is embedded in a solid host which acts as a reservoir of energy and of electrons. The host is characterized by a temperature T and electron chemical potential μ. Assume only the lowest ($1s$) hydrogen orbital can be occupied (*i.e.*, the average occupancy of the excited hydrogen levels is negligible for this problem). Denote the energy of the $1s$ state by ε. Two electrons in this orbital experience a Coulomb repulsion of energy U.

(a) List all the possible states of the hydrogen atom together with their corresponding energy levels. You may ignore the spin of the proton.

(b) Find the average number of electrons bound to the atom. For what relation between ε, μ, U, and T is the average number 1?

(c) Since electrons have a magnetic moment, a magnetic field H will induce a magnetization $M(T, H)$ for the atom. Find the magnetic susceptibility $\chi(T) = (\partial M/\partial H)_{H=0}$. Does increasing the Coulomb repulsion, U, increase or decrease the susceptibility?

10

Identification Number: 59

Problem 8

Consider the integral expression

$$J(a) = \frac{4a}{\sqrt{\pi}} \int_0^\infty dx \, \frac{x\, e^{-x^2}}{1 - e^{-a/x}}$$

(a) What is the limiting asymptotic form of $J(a)$ as $a \to 0$? Keep at least two terms in the expansion.

(b) What is the limiting asymptotic form of $J(a)$ as $a \to \infty$? Keep at least two terms in the expansion.

1

DEPARTMENT OF PHYSICS
UNIVERSITY OF CALIFORNIA AT SAN DIEGO
LA JOLLA, CALIFORNIA 92093-0354

WRITTEN DEPARTMENTAL EXAMINATION, FALL 1992

PART II

Identification Number: 59

Instructions

Each problem is worth 10 points. Part II has 7 problems (numbers 9 through 15).

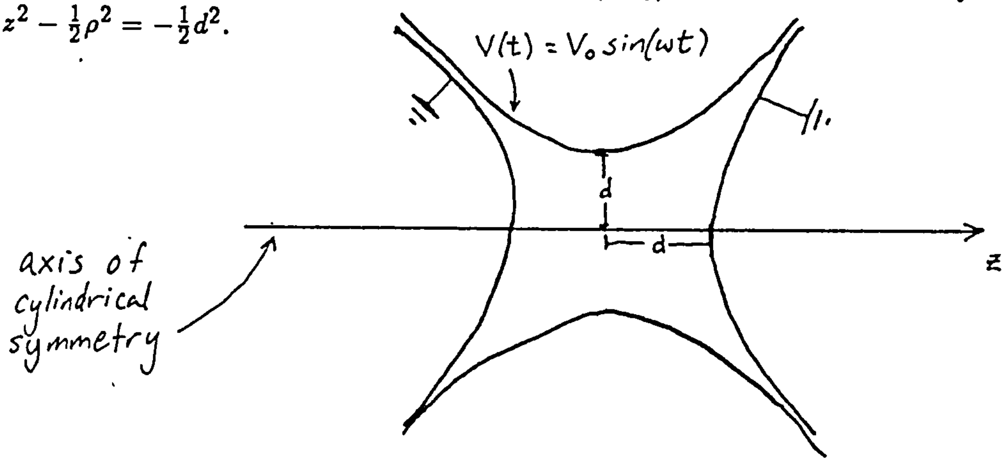
UCSD Physics Department Written Exam

Fall 1992 - Part II

Identification Number: 59

Problem 9

A Paul trap is used to confine charged particles. The trap is cylindrically symmetric, so it is convenient to describe its geometry in a cylindrical coordinate system (ρ, θ, z), where ρ is the radial distance and θ the azimuthal angle. Typically, the trap electrodes are hyperbolae of revolution (see figure below). The two end electrodes are defined by the equation $z^2 - \frac{1}{2}\rho^2 = d^2$, and the center (ring) electrode is defined by the equation $z^2 - \frac{1}{2}\rho^2 = -\frac{1}{2}d^2$.



(a) Suppose that the end electrodes are grounded and that the potential on the center electrode is given by $V(t) = V_0 \sin(\omega t)$, where $\omega \ll c/d$ (c is the speed of light). Determine the electric field inside the trap. *Hint:* $\nabla^2(z^2 - \frac{1}{2}\rho^2) = 0$.

(b) Under the condition that $\omega^2 \gg eV_0/md^2$, separate the motion of the electron into slow and fast components. Calculate the characteristic frequencies for the slow components of both the axial and radial motion.

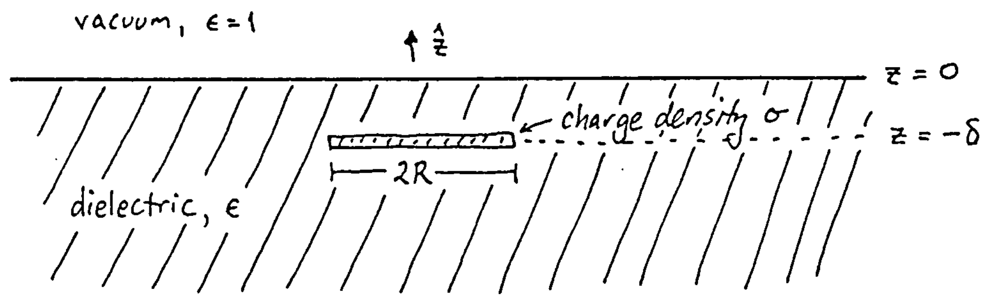
UCSD Physics Department Written Exam

Fall 1992 – Part II

Identification Number: 59

Problem 10

Consider a uniformly charged disc of radius R with two-dimensional charge density σ. The disc is embedded inside a semi-infinite dielectric medium with dielectric constant ϵ, as shown below. The disc lies along the plane $z = -\delta$, parallel to the vacuum-dielectric interface at $z = 0$.



(a) Find an expression for the electric potential $\Phi(r)$ everywhere in space.

(b) Find an expression for the total force on the dielectric. What is the direction of this force?

see p 92, 93 Jackson

9

UCSD Physics Department Written Exam

Fall 1992 - Part II

Identification Number: 59

Problem 11

An electron in a Coulomb potential is in the state $|n=2, l=1, j=\frac{1}{2}\rangle$. (You may *not* neglect spin-orbit coupling.) A time-dependent potential,

$$\begin{aligned} H'(t) &= 0 \qquad\qquad (t<0) \\ &= \alpha \boldsymbol{S}\cdot\boldsymbol{r}\, e^{-\gamma t} \qquad (t>0) \end{aligned}$$

with $\gamma > 0$, is now added. In this problem, you may work to lowest order in perturbation theory, and you may leave your answers in terms of radial integrals. Denote the initial and final radial wave functions by $R_{\rm i}(r)$ and $R_{\rm f}(r)$, respectively, and the energy difference as $\Delta E = E_{\rm f} - E_{\rm i}$. Useful information:

$$Y_{0,0} = \sqrt{\frac{1}{4\pi}} \qquad Y_{1,0} = \sqrt{\frac{3}{4\pi}}\cos\theta \qquad Y_{1,\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{\pm i\varphi}$$

(a) Find the probability of finding this system in its ground state after a long time.

(b) Repeat part (a), assuming the initial state had $j = \frac{3}{2}$.

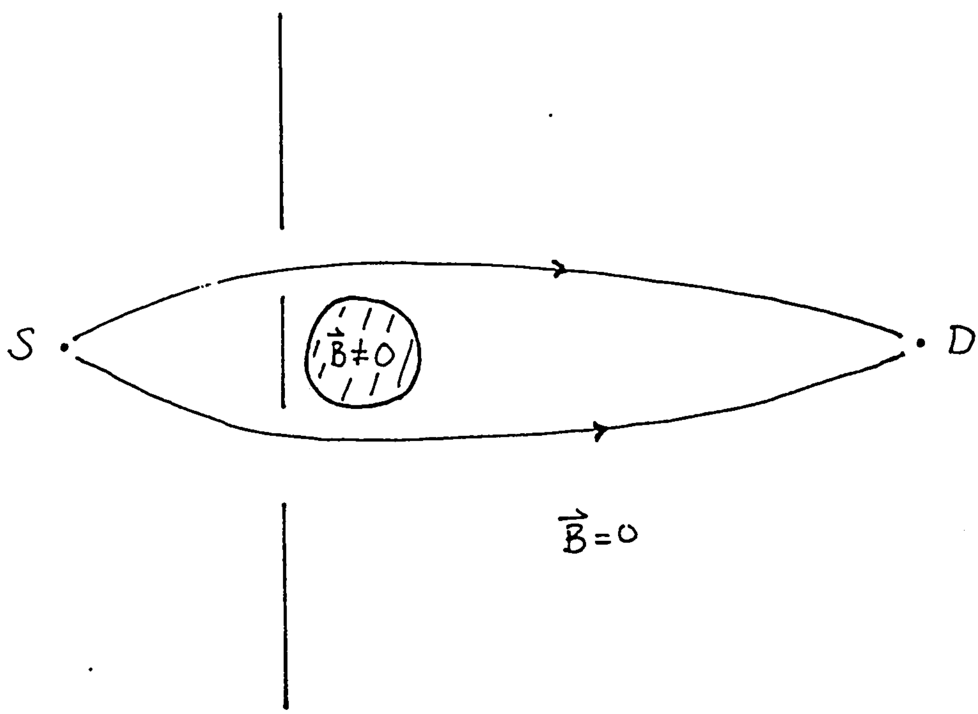
UCSD Physics Department Written Exam

Fall 1992 - Part II

Identification Number: 59

Problem 12

Consider the double slit geometry depicted below. Quantum mechanical particles of charge e originate from a source at S and are detected at point D. An infinitely long impenetrable cylindrical solenoid is positioned as shown. The magnetic field B in the solenoid is oriented parallel to its axis, normal to the plane of the figure. Show that the probability of detecting a particle at D oscillates ~~sinusoidally~~ periodically with the field strength B. What is the period in B, expressed in terms of physical constants and the cross-sectional area of the solenoid?



UCSD Physics Department Written Exam

Fall 1992 – Part II

Identification Number: 59

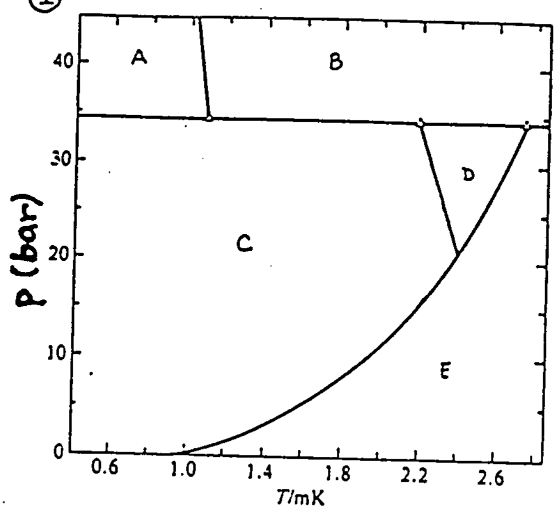
Problem 13

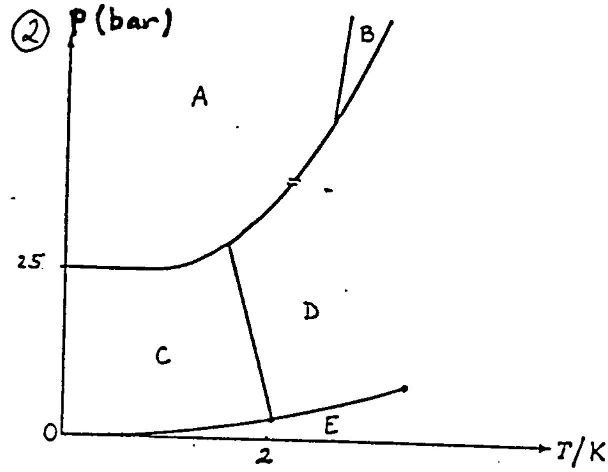
Consider an ideal gas of classical monatomic particles in the presence of a surface. The surface consists of N adsorption sites, each of which can accomodate at most one atom. The adsorbed atoms are bound to the sites with energy $-W$, where $W > 0$. The gas is at density n and temperature T and is in equilibrium with the surface. What fraction of adsorption sites is occupied?

UCSD Physics Department Written Exam

Fall 1992 – Part II

Problem 14 Identification Number: 59





Pictured above are two phase diagrams. One is for ^{4}He and the other for ^{3}He. Note the difference in scale of the temperature axes. In parts (a) and (b), you are expected to produce crude qualitative arguments based on sound physical reasoning. *Hint:* It may be useful to read this problem through before attempting to answer. For example, some of the options listed in (b) may help you in answering (a).

(a) Which is the phase diagram for ^{4}He? For this diagram, match the phase labels A through E with the following: gas, normal fluid, superfluid, solid, solid. (There are two solid phases which differ in crystal structure. In fact, there is even another solid phase not shown in the diagram.) Give a rough justification for your phase identifications.

(b) Which is the phase diagram for ^{3}He? For this diagram, match the phase labels A through E with the following: antiferromagnetic solid, paramagnetic solid, normal fluid, superfluid, superfluid. (There are two superfluid phases which differ in the nature of their order parameters.) Give a rough justification for your phase identifications.

(c) Chemically, these two helium isotopes are virtually identical. Both are neutral atoms with a filled electronic 1s shell. Why do they behave so differently at low temperature? Why does one have magnetic phases while the other does not?

(d) What is the (approximate) formula for the Bose condensation temperature T_c of a gas of ideal bosons of mass m at fixed number density n? Estimate T_c for liquid ^{4}He, assuming a mass density of $0.14\,\mathrm{g/cm^3}$.

UCSD Physics Department Written Exam

Fall 1992 - Part II

Identification Number: 59

Problem 15

Let $f(z)$ be analytic in the upper half plane, with $f(z) \to 0$ as $|z| \to \infty$. We can write $f(z) = U(x,y) + iV(x,y)$ where U and V are real and $z = x + iy$. If $U(x,0) = \frac{1}{1+|x|}$, find $V(1,0)$.

Problem 1
59

1 (a) We can use uncertainty principle:

The neutron is confined in a region

$$\Delta x \sim 1F = 10^{-15} m$$

$$\Rightarrow \quad \Delta p \sim \frac{10^{-15} m}{\hbar} \qquad \rightarrow p \sim \frac{\hbar}{\Delta x} \sim \frac{\hbar}{10^{-15}}$$

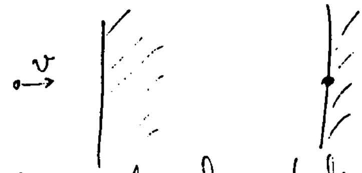
The speed must be of order $\frac{\Delta p}{\cancel{M_N}} \sim \boxed{\frac{10^{-15} m}{\cancel{M_N}\hbar}}^{-1}$

b) The average velocity:

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} KT \Rightarrow \langle v \rangle = \left(\frac{3KT}{m}\right)^{1/2}$$

We can compute the average # of molecule that incide on a unit area of the surface per unit time (dn/dt)

$$P = m \langle v \rangle \frac{dn}{dt}$$



the molecule colides and sticks in the surface $\Delta p = mv$

$$\boxed{\frac{dn}{dt} \cong \frac{P}{(3mKT)^{1/2}}}$$

Problem 2

59

the kinetic energy.

$$T = \frac{1}{2} M\dot{x}^2 + \frac{m}{2}\left(\dot{x}^2 + 2l\dot{x}\dot{\theta}\cos\theta + l^2\dot{\theta}^2\right)$$ ✓

the potential:

$$V = -mgl\cos\theta$$ ✓

$$\begin{aligned} L &= T - V \\ &= \frac{M}{2}\dot{x}^2 + \frac{m}{2}\dot{x}^2 + ml\dot{x}\dot{\theta}\cos\theta + \frac{m}{2}l^2\dot{\theta}^2 + mgl\cos\theta \end{aligned}$$

the deviation from the equilibrium position $\theta=0$ will be very small ($\dot{x}$ and $\dot{\theta}$ will be $O(\theta)$). So, up to $O(\theta^3)$ we have

$$L = \frac{M}{2}\dot{x}^2 + \frac{m}{2}\dot{x}^2 + ml\dot{x}\dot{\theta} + \frac{ml^2}{2}\dot{\theta}^2 + mgl - mgl\frac{\theta^2}{2}$$

$$\frac{d}{dt}\left(M\dot{x} + m\dot{x} + ml\dot{\theta}\right) = 0$$

$$\frac{d}{dt}\left(ml\dot{x} + ml^2\dot{\theta}\right) = -mgl\theta$$

$$\left\{\begin{aligned} &(M+m)\dot{x} + ml\dot{\theta} = 0 \quad \text{✓ (from initial condition)} \\ &\ddot{x} + l\ddot{\theta} = -g\theta \quad ✓ \end{aligned}\right.$$

Problem 2

\# 59

$$\ddot{\theta}\left(1-\frac{m}{M+m}\right)=-\frac{g}{\ell}\theta$$

$$\ddot{\theta}=-\left(\frac{M+m}{M}\right)\frac{g}{\ell}\theta$$

$$\boxed{\Omega^2=\frac{M+m}{M}\frac{g}{\ell}}$$

$$\boxed{\theta(t)=\theta_0\cos(\Omega t)}$$

From $\dot{x}=-\left(\frac{m}{M+m}\right)\ell\,\dot{\theta}$ $\quad x(0)=x_0 \quad \dot{x}(0)=0$

$$x(t)-x_0=-\left(\frac{m}{M+m}\right)\ell\left[\theta_0\cos(\Omega t)-\theta_0\right]$$

$$\boxed{x(t)=x_0+\left(\frac{m}{M+m}\right)\ell\theta_0\left[1-\cos\Omega t\right]}$$

$$\boxed{\Omega=\left(\frac{M+m}{M}\right)^{1/2}\left(\frac{g}{\ell}\right)^{1/2}}$$

~~[illegible]~~

Problem 3

\# 59

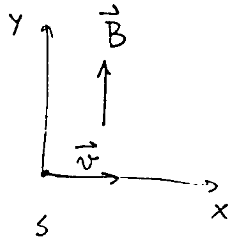
a) Neglecting relativistic effects (i.e. to $\theta(\frac{v}{c})$) we have an electric field $\vec{E}' = \frac{\vec{v}}{c} \times \vec{B}$ in the reference frame in which the sphere is at rest.

(This is to ensure that the Lorentz force in both frames be the same)

$$q \, \frac{\vec{v}}{c} \times \vec{B} = q \, \vec{E}'$$

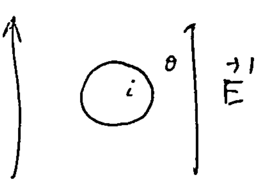
$$\boxed{\vec{E}' = \frac{\vec{v}}{c} \times \vec{B}}$$

✓



$$\Rightarrow \quad \vec{E}' = \frac{v}{c} B \hat{z}$$

A conductor sphere in a constant eletric field.



$\varphi_i = C_i$ (conductor!)

$$\varphi_o = -\frac{vB}{c} r \cos\theta + \frac{A}{r^2} \cos\theta$$

$$C_i = \left(-\frac{vBR}{c} + \frac{A}{R^2}\right) \cos\theta \qquad \varphi \text{ continuous}$$

$$\Rightarrow \quad C_i = 0 \Rightarrow \boxed{A = \frac{vB}{c} R^3}$$

(E_t is automatically continuous)

Problems 3

59

So we have an induced dipole

V

$$\vec{P} = \frac{vB}{c} R^3 \hat{z}$$

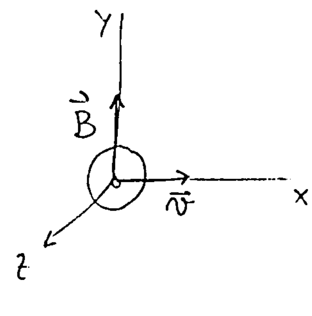
dipole field.

$$\begin{cases} \vec{E}' = \dfrac{vB}{c}\hat{z} + \dfrac{3(\vec{P}\cdot\hat{n})\hat{n} - \vec{P}}{r^3} & (r>R) \\ \quad = 0 & (r<R) \end{cases}$$

using again $\vec{E}' = \vec{E} + \frac{\vec{v}}{c}\times\vec{B}$ 5

we obtain an dipole field in the unprimed frame

$$\boxed{\vec{E} = \left(\frac{vBR^3}{c}\right)\frac{3(\hat{z}\cdot\hat{n})\hat{n} - \hat{z}}{r^3}} \quad (r>R)$$



and a constant field inside the sphere

$$\boxed{\vec{E} = -\frac{vB}{c}\hat{z}} \quad (r<R)$$

b) since $\operatorname{div}\vec{E} = 0$, inside we must have $\boxed{\rho = 0}$ V

on the surface 5

$$E_r^+ - E_r^- = 4\pi\sigma$$

$$\boxed{\sigma(\theta) = \frac{3}{4\pi}\left(\frac{vB}{c}\right)\cos\theta}$$

$$\left(\frac{vB}{c}\right)\left[3\cos\theta - \cos\theta\right] + \frac{vB}{c}\cos\theta = 4\pi\,\sigma(\theta)$$

Problem 4
59

a) the hidrogen ground state energy:

$$E_H = -\frac{m e^4}{2\hbar^2} = -13.6\ eV$$

where m is the reduced mass

$$\frac{1}{m} = \frac{1}{m_p} + \frac{1}{m_e} \Rightarrow m \sim m_e$$

2

for the positronium every thing is the same but

$$\frac{1}{m} = \frac{2}{m_e} \Rightarrow m = \frac{m_e}{2}$$

hence for the postromium

$$\boxed{E_P = \frac{E_H}{2} = -6.8\ eV}$$

b) Since $[\vec{J}, H] = 0$ the rotations comute with H $[D(R), H] = 0$ and rotated states have the same energy.

Since $x\phi(r)$ is $z\phi(r)$ rotated it is an eigenstate with eigenvalue E.

Problem 6

59

a) the work done by the bar to extend it by $d\xi$

$$dW = -\tau(\xi, T)\, d\xi$$

$$dU = T\,dS - dW \qquad \text{(first law)}$$

$$\boxed{dU = T\,dS + \tau\, d\xi}$$ ✓

b) Consider $F = U - ST$

$$dF = -S\,dT + \tau\, d\xi$$

equality of mixed partials give:

$$\boxed{-\left(\frac{\partial S}{\partial \xi}\right)_T = \left(\frac{\partial \tau}{\partial T}\right)_\xi}$$ ✓

(c) we write

$$dU = T\left(\frac{\partial S}{\partial \xi}\right)_T d\xi + T\left(\frac{\partial S}{\partial T}\right)_\xi \overset{0}{dT} + \tau\, d\xi$$

$$dU)_T = \left[-T\left(\frac{\partial \tau}{\partial T}\right)_\xi + \tau\right] d\xi$$

Problem 6

59

$$\left.\frac{\partial \tau}{\partial T}\right)_\xi = -\, b \xi \gamma$$

$$dU)_T = \left[b\xi\gamma T + b\xi - b\xi\gamma T \right] d\xi$$

$$= b\xi \, d\xi$$

hence

$$\Delta U)_T = \int_0^{\xi_0} b\xi \, d\xi$$

$$= \boxed{\frac{b\xi_0^2}{2}}$$

(d) The heat is given by

$$dQ)_T = T\left(\frac{\partial S}{\partial \xi}\right)_T d\xi$$

$$= -T\left(\frac{\partial \tau}{\partial T}\right)_\xi d\xi$$

$$= b\xi\gamma T \, d\xi$$

$$\Delta Q)_T = b\gamma T \int_0^{\xi_0} \xi \, d\xi$$

$$\boxed{= \frac{b\gamma T \xi_0^2}{2}}$$

Heat ABSORBED

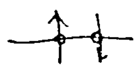
Problem 7

\# 59

a) We have two possibilities with one electron in the (1s) state

energy ε

with 2 electrons in the same orbital the total spin has to be zero



singlet

energy $2\varepsilon + U$

We have 3 possibilities:

$$\underset{(1s)\uparrow}{\varepsilon}\,,\quad \underset{(1s)\downarrow}{\varepsilon}\,,\quad \underset{(1s)^2 S}{2\varepsilon + U}$$

singlet

3

b) With this we construct the gran partition function

$$\mathcal{Z} = 2 e^{\beta\mu} e^{-\beta\varepsilon} + e^{\beta\mu 2} e^{-\beta(2\varepsilon+U)} = e^{\beta(\mu-\varepsilon)}\left[2 + e^{\beta(\mu-\varepsilon-U)}\right]$$

$$\langle N\rangle = \frac{1}{\beta}\frac{\partial}{\partial\mu}\ln \mathcal{Z}$$

$$\langle N\rangle = \frac{\sum N_k e^{-\beta(E_k - \mu N_k)}}{\sum e^{-\beta(E_k - \mu N_k)}}$$

$$= \frac{1}{\beta}\frac{\partial}{\partial\mu}\left[\beta(\mu-\varepsilon) + \ln\left(2 + e^{\beta(\mu-\varepsilon-U)}\right)\right]$$

$$= 1 + \frac{e^{\beta(\mu-\varepsilon-U)}}{2 + e^{\beta(\mu-\varepsilon-U)}}$$

Problem 7

59

When

$$\boxed{\frac{\mathcal{E}+U-\mu}{KT} \gg 1}$$

$\langle N \rangle \to 1$.

2) (in the other limit $\frac{\mu-\mathcal{E}-U}{KT} \gg 1$ $\langle N \rangle \to 2$)

(c) the energy of the 3 states in the presence of the B-field.

- $|\uparrow\rangle \longrightarrow \mathcal{E} - mB$
- $|\downarrow\rangle \longrightarrow \mathcal{E} + mB$
- $|\text{sing}\rangle \longrightarrow 2\mathcal{E} + U$ $(S_z = 0)$

again:

$$\mathcal{Z} = e^{\beta\mu}\left[e^{-\beta\mathcal{E}} e^{\beta mB} + e^{-\beta\mathcal{E}} e^{-\beta mB} + e^{\beta\mu} e^{-\beta 2\mathcal{E}} e^{-\beta U} \right]$$

$$= e^{\beta(\mu-\mathcal{E})}\left[2\cosh(\beta mB) + e^{\beta(\mu-\mathcal{E}-U)} \right]$$

magnetization along B

$$M = \frac{1}{\beta}\frac{\partial \ln \mathcal{Z}}{\partial B} = \frac{1}{\beta}\frac{\partial}{\partial B}\left[\beta(\mu-\mathcal{E}) + \ln\left[2\cosh(\beta mB) + e^{\beta(\mu-\mathcal{E}-U)}\right] \right]$$

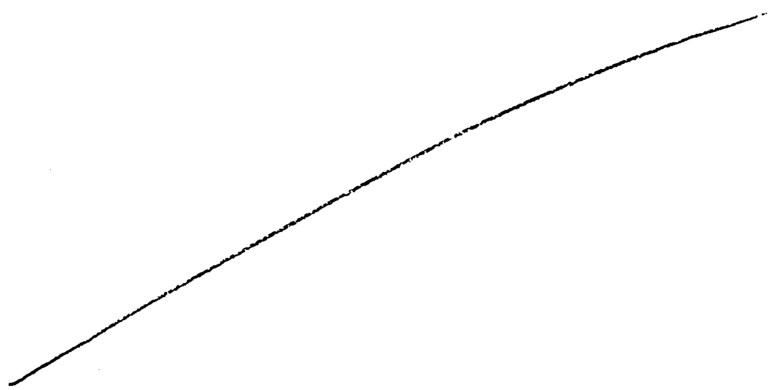
$$= \frac{1}{\cancel{\beta}} \frac{2\sinh(\beta mB)}{2\cosh(\beta mB) + e^{\beta(\mu-\mathcal{E}-U)}} (\cancel{\beta} m)$$

Problems 4
#59

The same cannot be said about $r\phi(r)$: This wave function cannot be obtained as a linear combination of $z\phi$, $y\phi$ and $x\phi$ in the subspace of eigenenergy E. Hence it does not necessarily have energy E.

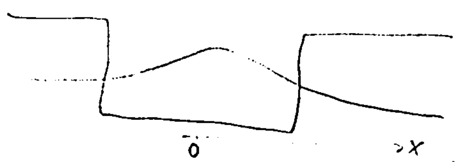
2

(c)



6

(d) We had



The ground state is even. The introduction of the impenetrable wall demands

$$\psi(0) = 0.$$

The ground state is still an eigenstate of H on the RHS of the well but it does not obey the boundary condition. We must have at least an excited state (odd) ↑ $\psi(0)=0$

in order to have a bound state for the second case

2

NO BOUND STATE IN THE NEW WELL

(e) We can argue as follows.

Due to virial theorem, for the H.O.

$$\langle T \rangle = \langle V \rangle$$

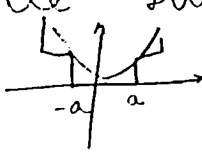
$$\rightarrow \quad E = 2\langle T \rangle$$

in the well $\quad E = \langle T \rangle$

Both ground states being confined in the same region Δx

we must have by uncertainty principle

$\langle \Delta p \rangle$ and hence $\langle T \rangle$ must be of the same order $\Rightarrow$ the ground state of the well is lower

we can also argue that since a is not specified 

Problem 5

#59

c) If the particles are identical and have spin ½ the wave function must be antisymmetrical. Only the state $|00\rangle$ is allowed

ground state energy: $\boxed{\frac{3}{4} J\hbar^2}$

$$\boxed{|00\rangle = \frac{1}{\sqrt{2}}\left[|+-\rangle - |-+\rangle\right]}$$

$|10\rangle = \frac{1}{\sqrt{2}}\left[|+-\rangle + |-+\rangle\right.$

$|11\rangle = |++\rangle$

$|1-1\rangle = |--\rangle$

For small B we expand.

$$M \sim m \frac{2\beta m B}{2 + e^{\beta(\mu-\varepsilon-U)}} .$$

Hence

$$\chi(T) = \left(\frac{\partial M}{\partial H}\right)_{H=0} = \boxed{\left(\frac{m^2}{KT}\right)\frac{1}{1+\frac{e^{\beta(\mu-\varepsilon-U)}}{2}}}$$

$\boxed{\text{increasing } U \text{ increases } \chi}$

$\chi \to \frac{m^2}{KT}$ from below

Problem 5

59

$$H \begin{Bmatrix} |11\rangle \\ |10\rangle \\ |1\bar{1}\rangle \end{Bmatrix} = -\frac{J\hbar^2}{4} \begin{Bmatrix} |11\rangle \\ |10\rangle \\ |1\bar{1}\rangle \end{Bmatrix}$$

$$H\,|00\rangle = \frac{3}{4} J\hbar^2 \,|00\rangle$$

Very good!

all 4 states are valid since the particles being distinguishable we have no constrain on the symmetry of the wave functions.

(b)

common magnetic moment.

$$H = -\frac{J}{2}\left(S^2 - \frac{3}{2}\hbar^2 \mathbb{1}\right) - \mu B \frac{2}{\hbar}\left(S_1^z + S_2^z\right)$$

again the 4 states $|S\, m_S\rangle$ are eigenstates:

$$H\,|11\rangle = \left(-\frac{J\hbar^2}{4} - \mu B\right)|11\rangle$$

$$H\,|10\rangle = \left(-\frac{J\hbar^2}{4}\right)|10\rangle$$

$$H\,|1\bar{1}\rangle = \left(-\frac{J\hbar^2}{4} + \mu B\right)|1\bar{1}\rangle$$

$$H\,|00\rangle = \left(\frac{3}{4} J\hbar^2\right)|00\rangle$$

again no symmetry requirement is made about the wave functions.

Problem 8

#59

(a) We use. $\boxed{a \sim 0}$

$$(1 - e^{-a/x})^{-1} = \left(\frac{a}{x} - \frac{a^2}{2x^2} + \frac{a^3}{6x^3} + \cdots \right)^{-1}$$

$$= \frac{x}{a} \left[1 - \frac{a}{2x} + \frac{a^2}{6x^2} \cdots \right]^{-1}$$

$$= \frac{x}{a} \left[1 + \frac{a}{2x} + \cdots \right]$$

we have then:

$$J(a) \sim \frac{4}{\sqrt{\pi}} \int_0^\infty dx \left(1 + \frac{a}{2x}\right) x^2 e^{-x^2}$$

$$\sim \frac{4}{\sqrt{\pi}} \left[\underbrace{\int_0^\infty x^2 e^{-x^2} dx}_{\frac{\sqrt{\pi}}{4}} + \frac{a}{2} \underbrace{\int_0^\infty x e^{-x^2} dx}_{1/2} \right]$$

$$\sim \boxed{1 + \frac{a}{\sqrt{\pi}}} \quad \boxed{a \to 0}$$

Problem 8

(b) $a \to \infty$ # 59.

$$e^{-a/x} \ll 1$$

$$(1 - e^{-a/x})^{-1} \sim 1 + e^{-a/x}.$$

Hence

$$J(a) \approx \frac{4a}{\sqrt{\pi}} \Big[\underbrace{\int_0^\infty x e^{-x^2} dx}_{1/2} + \int_0^\infty dx\, x\, e^{-(x^2 + a/x)}$$

We use saddle point in the second integral.

$$f(x) = x^2 + \frac{a}{x}$$

$$f' = 2x - \frac{a}{x^2} \qquad \Rightarrow x_0 = \left(\frac{a}{2}\right)^{1/3}.$$

$$f'' = 2 + \frac{2a}{x^3}$$

$$f(x_0) = \left(\frac{a}{2}\right)^{2/3} + 2^{1/3} a^{2/3} = a^{2/3}\left(\frac{1}{2^{2/3}} + 2^{1/3}\right) = \frac{3}{4^{1/3}} a^{2/3}$$

$$f''(x_0) = 6$$

We have then

$$\int_0^\infty dx\, x\, e^{-(x^2 + a/x)} \sim \int_0^\infty dx\, x_0\, e^{-f(x_0)} e^{-f''(x_0)\frac{(x-x_0)^2}{2}}$$

$$\simeq x_0 e^{-f(x_0)} \left(\frac{2\pi}{f''(x_0)}\right)^{1/2}$$

Problem 8

59

we have then: $a \to \infty$

$$\boxed{J(a) \sim \frac{4a}{\sqrt{\pi}}\left[\frac{1}{2} + \left(\frac{a}{2}\right)^{1/3}\left(\frac{\pi}{3}\right)^{1/2} \exp\left(-\frac{3}{4^{1/3}}a^{2/3}\right)\right]}$$

good

Problem 9

#59

(a) We can use quasi static approx, at each t the field is the eletrostatic field with the given boundary conditions AT THE SAME TIME (no radiation).

By symetry $\varphi = \varphi(z,\rho)$

$$\nabla^2 \varphi = 0$$

we can try $\varphi(z,\rho) = \left(z^2 - \frac{1}{2}\rho^2 - d^2\right)A$

$\nabla^2\varphi = 0$ and the b.c. at the grounded electrodes are satisfied.

To satisfy the b.c. at the ring we require:

$$-\frac{3}{2}d^2 A = V_0 \sin\omega t$$

$$A = -\frac{2}{3}\frac{V_0}{d^2}\sin\omega t$$

Finally:

$$\boxed{\varphi(z,\rho) = -\frac{2}{3}\frac{V_0}{d^2}\sin\omega t\left(z^2 - \frac{\rho^2}{2} - d^2\right)}$$

The electric field: $\vec{E} = -\vec{\nabla}\varphi = -\left(\frac{\partial\varphi}{\partial\rho}\hat{\rho} + \frac{\partial\varphi}{\partial z}\hat{z}\right)$

$$\boxed{\vec{E}(z,\rho) = \left(\frac{2}{3}\frac{V_0}{d^2}\sin\omega t\right)\left(-\rho\hat{\rho} + 2z\hat{z}\right)}$$

5

Problem 9

\# 59

(b) $m\ddot{\vec{x}} = -e\vec{E}$ $(e>0)$

$$\vec{x} = \rho\hat{\rho} + z\hat{z}$$
$$\dot{\vec{x}} = \dot{\rho}\hat{\rho} + \rho\dot{\theta}\hat{\theta} + \dot{z}\hat{z}$$
$$\ddot{\vec{x}} = \ddot{\rho}\hat{\rho} + 2\dot{\rho}\dot{\theta}\hat{\theta} + \rho\ddot{\theta}\hat{\theta} - \rho\dot{\theta}^2\hat{\rho} + \ddot{z}\hat{z}$$
$$= (\ddot{\rho} - \rho\dot{\theta}^2)\hat{\rho} + (2\dot{\rho}\dot{\theta} + \rho\ddot{\theta})\hat{\theta} + \ddot{z}\hat{z}$$

due to the cylindrical symmetry we can use the conservation of L_z.

conserved angular momentum $\longrightarrow$ $\boxed{l = m\rho^2\dot{\theta}}$

$$\dot{\theta} = \frac{l}{m\rho^2} \qquad \ddot{\theta} = \frac{-2l}{m\rho^3}\dot{\rho}$$

$$\ddot{\vec{x}} = \left(\ddot{\rho} - \frac{l^2}{m^2\rho^3}\right)\hat{\rho} + \cancel{\left(\frac{2\dot{\rho}l}{m\rho^2} - \frac{2l\dot{\rho}}{m\rho^2}\right)}\hat{\theta} + \ddot{z}\hat{z}$$

we are left with:

$$\begin{cases} \ddot{\rho} - \dfrac{l^2}{m^2\rho^3} = \left(\dfrac{2}{3}\dfrac{V_0}{d^2}\dfrac{e}{m}\sin\omega t\right)\rho \\[2ex] \ddot{z} = \left(-\dfrac{4}{3}\dfrac{V_0}{d^2}\dfrac{e}{m}\sin\omega t\right)z \end{cases}$$

$$\left(\Omega^2 = \frac{V_0 e}{m d^2}\right) \qquad \Omega^2 \ll \omega^2$$

Problem 9

59

$$\begin{cases} \dfrac{\ddot{\rho}}{\rho} = \dfrac{l^2}{m^2\rho^4} + \dfrac{2}{3}\Omega^2 \sin\omega t \\[2ex] \dfrac{\ddot{z}}{z} = -\dfrac{4}{3}\Omega^2 \sin\omega t \end{cases}$$

the z movement is

$$\boxed{z(t) = \frac{4}{3}\frac{\Omega^2}{\omega^2}\sin\omega t}$$

this movement is slow since $\dot{z} \sim \frac{\Omega^2}{\omega}\cos\omega t \ll 1$ and has a very small amplitude.

For the radial motion we identify a slow term (small amplitude $\mathcal{O}(\frac{\Omega^2}{\omega^2})$) which is

$$\frac{\ddot{\rho}}{\rho} = \frac{2}{3}\Omega^2 \sin\omega t$$

neglecting such small oscillations we have up to $\mathcal{O}(\frac{\Omega^2}{\omega^2})$

$$\ddot{\rho} = \frac{l^2}{m^2\rho^3}$$

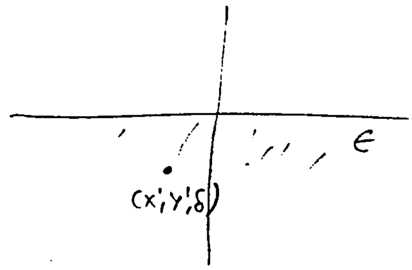
the solutions of this equation are the fast components of the motion.

the frequency is of order $\frac{\ddot{\rho}}{\rho} \sim \frac{l^2}{m^2 d^4} \Rightarrow \boxed{\omega_{F\,\text{fast}} \sim \frac{l}{md^2}}$

↳ while $\boxed{\omega_{s\,\text{slow}} = \omega}$

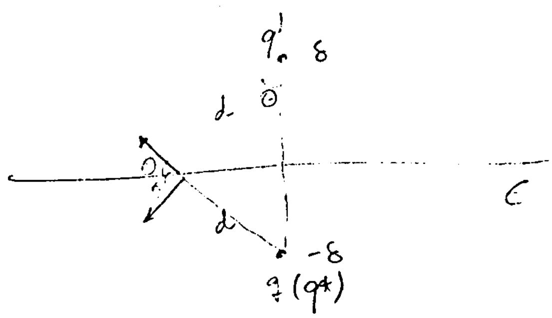
Problem 10
#59

(a) We can solve the problem of a unit charge at $(x', y', -\delta)$



and then, with $\phi(\vec{r}, x', y', -\delta)$, use the superposition principle to obtain

$$\phi_{Disc}(\vec{r}) = \int_{disc} \phi(\vec{r}\,|\,x', y, -\delta)\, \sigma\, da'$$



in the UHP : $E_t = \dfrac{q^*}{d^2}\cos\theta$

in the LHP $E_t = \dfrac{q'}{\epsilon d^2}\cos\theta + \dfrac{q}{\epsilon d^2}\cos\theta$

$$\boxed{q + q' = \epsilon q^*}$$

in the UHP $D_n = \dfrac{q^*}{d^2}\sin\theta$

LHP $D_n = \left(\dfrac{-q'}{d^2} + \dfrac{q}{d^2}\right)\sin\theta$

$$\boxed{q^* = q - q'}$$

we find that Problem 10

59

$$2q = (\epsilon+1)\,q^*$$

$$\boxed{q^* = \frac{2}{\epsilon+1}\,q}$$ effective charge for the UHP

$$q' = \left(\frac{2\epsilon}{\epsilon+1} - 1\right) q$$

$$\boxed{q' = \left(\frac{\epsilon-1}{\epsilon+1}\right) q}$$

the potential: $(q=1)$

UHP: $$\phi(\vec r\,|\,x',y',-\delta) = \frac{2}{\epsilon+1}\,\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z+\delta)^2}}$$

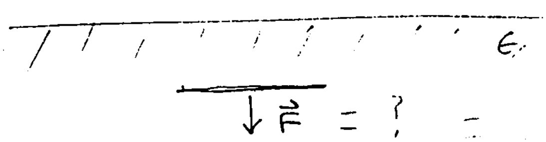
LHP: $$\phi(\vec r\,|\,x',y',-\delta) = \left(\frac{\epsilon-1}{\epsilon+1}\right)\frac{(1/\epsilon)}{\sqrt{(x-x')^2+(y-y')^2+(z-\delta)^2}} + \frac{(1/\epsilon)}{\sqrt{(x-x')^2+(y-y')^2+(z+\delta)^2}}$$

$$\boxed{\vec E = -\vec\nabla\varphi}$$

$$\phi(r) = \int_{x',y'\in \text{disc}} \phi(r\,|\,x',y',-\delta)\,\sigma\,dx'dy'$$

(b) From the problem of a point charge we found that the charge inside the dielectric sees a charge $q' = \left(\frac{\epsilon - 1}{\epsilon + 1}\right) q$ right above it at $z = 8$

the charges having the same sign the force is repulsive. In the case of the disc, by symetry, the force has to be along the z-direction



#59

Problem. 11

We have the final state in interaction picture

$|f\rangle_I$, we want to compute

$$P = |{}_I\langle f|\, U_I(\infty,0)\,|i\rangle_I|^2$$

$$= |\langle f|\, U_I(\infty,0)\,|i\rangle|^2$$ (the pases $e^{iE_i t/\hbar}$, $e^{iE_f t/\hbar}$ drop out)

U_I in first order perturbation theory.

$$U_I(t,0) = \hat{1} + \frac{1}{i\hbar}\int_0^t e^{-iH_0 t'/\hbar}\, V(t')\, e^{iH_0 t'/\hbar}\, dt'$$

$$\langle f|U_I|i\rangle = \frac{1}{i\hbar}\int_0^t e^{-i\Delta E t'/\hbar}\, \alpha\, \langle f|\vec{S}\cdot\vec{r}|i\rangle\, e^{-\gamma t'}\, dt'$$

where we've used $\langle i|f\rangle = \langle n=0, l=0, j=1/2 | n=2, l=1, j=1/2\rangle = 0$

$$\langle f|U_I(\infty,0)|i\rangle = \frac{1}{i\hbar}\,\alpha\,\langle f|\vec{S}\cdot\vec{r}|i\rangle\,\frac{1}{\left(\gamma + i\frac{\Delta E}{\hbar}\right)}$$

$$P = \left(\frac{\alpha^2}{\hbar^2\gamma^2 + \Delta E^2}\right)|\langle f|\vec{S}\cdot\vec{r}|i\rangle|^2$$

2

problem 11

We now have to change basis from

$|m_\ell\, m_s\rangle$ to $|j\, m\rangle$ $\quad (J_- = L_- + S_-)$

$$\boxed{|1\ \tfrac{1}{2}\rangle = |\tfrac{3}{2}\ \tfrac{3}{2}\rangle}$$

$$\sqrt{2}\,|0\ \tfrac{1}{2}\rangle + |1\ -\tfrac{1}{2}\rangle = \sqrt{3}\,|\tfrac{3}{2}\ \tfrac{1}{2}\rangle$$

$$\boxed{\frac{\sqrt{2}}{\sqrt{3}}|0\ \tfrac{1}{2}\rangle + \frac{1}{\sqrt{3}}|1\ -\tfrac{1}{2}\rangle = |\tfrac{3}{2}\ \tfrac{1}{2}\rangle} \xrightarrow{\text{ortog}} \boxed{\frac{1}{\sqrt{3}}|0\ \tfrac{1}{2}\rangle - \frac{\sqrt{2}}{\sqrt{3}}|1\ \bar{\tfrac{1}{2}}\rangle = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle}$$

$$\frac{\sqrt{2}}{\sqrt{3}}\sqrt{2}\,|\bar{1}\ \tfrac{1}{2}\rangle + \left(\frac{\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{2}}{\sqrt{3}}\right)|0\ \bar{\tfrac{1}{2}}\rangle = \sqrt{4}\,|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle$$

$$\boxed{\frac{1}{\sqrt{3}}|\bar{1}\ \tfrac{1}{2}\rangle + \frac{\sqrt{2}}{\sqrt{3}}|0\ \bar{\tfrac{1}{2}}\rangle = |\tfrac{3}{2}\ -\tfrac{1}{2}\rangle} \xrightarrow{\text{ortog}} \boxed{\frac{\sqrt{2}}{\sqrt{3}}|\bar{1}\ \tfrac{1}{2}\rangle - \frac{1}{\sqrt{3}}|0\ \bar{\tfrac{1}{2}}\rangle = |\tfrac{1}{2}\ \bar{\tfrac{1}{2}}\rangle}$$

$$\boxed{|\bar{1}\ \bar{\tfrac{1}{2}}\rangle = |\tfrac{3}{2}\ -\tfrac{3}{2}\rangle}$$

the initial state can be either $|\tfrac{1}{2}\ \tfrac{1}{2}\rangle$ and $|\tfrac{1}{2}\ \bar{\tfrac{1}{2}}\rangle$

since $\vec{S}\cdot\vec{r}$ is a scalar, Wigner-Eckart states that it has non-zero matrix element provided $\boxed{\Delta m = \Delta j = 0}$.

$$|n=2, \ell=1, j=\tfrac{1}{2}, \pm\rangle \longrightarrow |n=0, \ell=0, j=\tfrac{1}{2}, \pm\rangle$$

note that parity is ok since $\vec{S}\cdot\vec{r}$ is odd and $|i\rangle$ and $|f\rangle$ has opposite parities. $\boxed{(-)^\ell}$

#59

problem 11

Let's compute the matrix element:

$$\vec{S}\cdot\vec{r} = \frac{\hbar}{2}\begin{bmatrix} z & x-iy \\ x+iy & -z \end{bmatrix}$$

$$|f\rangle = |n=0,\ l=0,\ j=1/2,\ m=1/2\rangle$$

$$\langle r|f\rangle = \begin{bmatrix} R_f(r)\, Y_{00} \\ 0 \end{bmatrix}$$

$$|i\rangle = |n=2,\ l=1,\ j=1/2,\ m=1/2\rangle$$

$$\langle r|i\rangle = \frac{R_i(r)}{\sqrt{3}}\begin{bmatrix} Y_{10} \\ -\sqrt{2}\, Y_{11} \end{bmatrix}$$

$$\langle f|\vec{S}\cdot\vec{r}|i\rangle = \int d^3r\ \frac{R_i(r)\, R_f^*(r)}{\sqrt{3}}\left(Y_{10}^*\, z\, Y_{00} - \sqrt{2}(x+iy)\, Y_{11}^*\, Y_{00}\right)$$

there are some cancellations left out

$$\int d\Omega\ Y_{10}^*\, z\, Y_{00} = \int d\Omega\left(\sqrt{\frac{3}{4\pi}}\cos\theta\right)(r\cos\theta)\sqrt{\frac{1}{4\pi}}$$

$$= \frac{\sqrt{3}}{4\pi}\, 2\pi\, r \int_{-1}^{1}\mu^2\, d\mu$$

$$= \frac{\sqrt{3}}{2}\, r\, \frac{2}{3} = \boxed{\frac{r}{\sqrt{3}}}$$

$$(-\sqrt{2}\, r)\int d\Omega\ (\sin\theta\, e^{i\varphi})\left(-\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{-i\varphi}\right)\sqrt{\frac{1}{4\pi}}$$

$$= \frac{\sqrt{6}\, r}{\sqrt{2}\, 4\pi}\, 2\pi \int_{-1}^{1}(1-\mu^2)\, d\mu = \frac{\sqrt{3}\, r}{2}\left(2-\frac{2}{3}\right) = \boxed{\frac{2}{\sqrt{3}}\, r}$$

We are left with

$$\boxed{\int_0^\infty dr\, r^3 R_i(r)\, R_f^*(r)}$$

result valid also for initial and final states with $m = -1/2$.

$$\boxed{P = \frac{\alpha^2}{\hbar^2\gamma^2 + \Delta E^2} \left| \int_0^\infty R_i(r) R_f^*(r)\, r^3\, dr \right|^2}$$

(b) Since $\vec{S}\cdot\vec{r}$ is a scalar it cannot have matrix elements with $\Delta J \neq 0$

$\boxed{P = 0}$ $\qquad [J, \vec{S}\cdot\vec{r}] = 0$

Problem 12

#59

We use Feynman path integral formalism.

$$\langle D,t \mid S,0\rangle = \int d(\text{all paths from } S \text{ to } D)\; e^{\frac{-iS(\text{path})}{\hbar}}$$

prob amplitude of detecting the particle at D at time t.

given a path $\vec{x}(t')$, $[\,\vec{x}(0)=S\ ,\ \vec{x}(t)=D\,]$ we have the action.

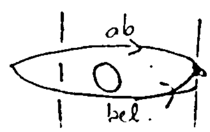
$$S[\vec{x}(t')] = \int_0^t \mathcal{L}_{free}[\vec{x}(t')] + \frac{q}{c}\dot{\vec{x}}\cdot\vec{A}\; dt'$$

$$= \int_0^t \mathcal{L}_f + \int_0^t \frac{q}{c}\frac{d\vec{x}}{dt'}\cdot\vec{A}\,dt'$$

$$= \int_0^t \underbrace{\mathcal{L}_f}_{\frac{1}{2}m\dot{\vec{x}}^2(t')} + \int_S^D \frac{q}{c}\vec{A}\cdot\vec{dl} \quad ✓$$

line integral of $\vec{A}$ along the path

We consider two kinds of path, the ones passing above and the ones bellow the solenoid



$\neq 33$

problem 12

we can pull out the $(1-e^{\phi})$ term to get for the probability:

$$|\langle D,t|S,0\rangle|^2 = \left|\sum_{\substack{\text{path}\\\text{above}}} e^{-i\frac{S_f}{\hbar}} e^{-i\frac{S_A}{\hbar}}\right|^2 \left|1-e^{-i\frac{qBA}{c\hbar}}\right|^2$$

we get a factor $\boxed{\sin^2\left(\frac{eBA}{2c\hbar}\right)}$ ~~mark~~.
$(q=e)$

the period in B is given by:

$$\frac{e(\Delta B)A}{2c\hbar} = \pi$$

$$\boxed{\Delta B = \frac{hc}{eA}} = \frac{\phi_0}{A}$$

x but ok.

#59

The fase factor introduced by the vector potential in the path "bellow" can be writte

$$\exp\left[-i\int_{S\,(\text{"below"})}^{D} \frac{q}{c}\frac{\vec{A}}{\hbar}\cdot d\vec{l}\right] = \exp\left[-i\int_{S\,(\text{above})}^{D} \frac{q}{c}\frac{\vec{A}}{\hbar}\cdot d\vec{l} - \oint i\frac{q}{c}\frac{\vec{A}}{\hbar}\cdot d\vec{l}\right]$$

$$= \exp\left[-\int_{\text{above}} i\frac{q}{c}\frac{\vec{A}}{\hbar}\cdot d\vec{l}\right] \exp -\frac{qBAi}{c\hbar}$$

where we've used

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

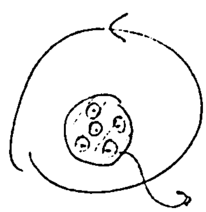
$$\oint \vec{B}\cdot\hat{n}\,dS = \oint \vec{A}\cdot d\vec{l}$$

area

$$B.A = \oint \vec{A}\cdot d\vec{l}$$

flux of $\vec{B}$

assume $\vec{B}$ poyting to your face



$$\oint \vec{A}\cdot d\vec{l} = B.A.$$

cross-sectional area = A

We have:

free, vector potential term

$$\langle D,t|S,0\rangle = \sum_{\text{path}} e^{-\frac{iS_f}{\hbar}} e^{-\frac{iS_A}{\hbar}}$$

combining path from above and bellow, with the same free action

$$= \sum_{\substack{\text{path}\\ \text{above}}} e^{-\frac{iS_f}{\hbar}} e^{-\frac{iS_A}{\hbar}} \left(1 + e^{-\frac{qBAi}{c\hbar}}\right)$$

problem 13

the grand partition function of the gas. (MB gas)

$$\ln Z = \sum_{l} e^{\beta(\mu - \varepsilon_l)}$$

$$= \frac{V}{h^3} \int e^{-\beta p^2/2m} \, d^3p \, (e^{\beta\mu})$$

$$= e^{\beta\mu} \frac{V}{\Lambda^3} \qquad \Lambda = \left(\frac{h^2}{2\pi m KT}\right)^{1/2}$$ thermal wavelenght.

$$\langle N \rangle = \frac{1}{\beta} \frac{\partial}{\partial \mu} \ln Z$$

$$= e^{\beta\mu} \frac{V}{\Lambda^3} \Rightarrow \boxed{e^{\beta\mu} = \Lambda^3(T)\, n}$$

For the sites;

$$Z = \sum_{l=0}^{N} e^{\beta(\mu+W)n} \, C_N^n .$$

$$= \left(1 + e^{\beta(\mu+W)}\right)^N$$

$$\ln Z = N \ln\left(1 + e^{\beta\mu} e^{\beta W}\right)$$

what we want to know

$$\langle N \rangle = \frac{1}{\beta} \frac{\partial}{\partial \mu} \ln Z = \frac{N}{\cancel{\beta}} \frac{\cancel{\beta} e^{\beta\mu} e^{\beta W}}{1 + e^{\beta\mu} e^{\beta W}}$$

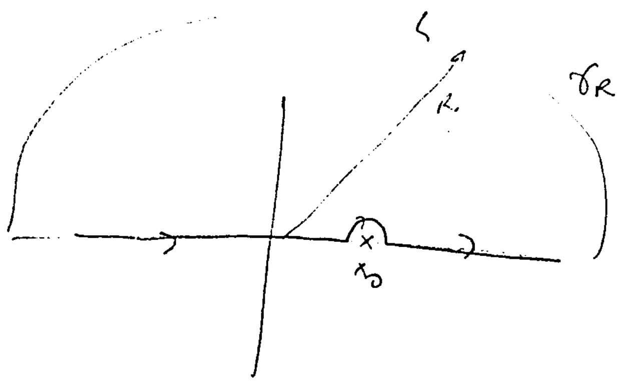
$$= N \left(\frac{1}{e^{-\beta\mu} e^{-\beta W} + 1}\right)$$

Problem 15

#59

We shall use the dispersion relations.

$$f(x_0,0) = \frac{1}{\pi i}\, P.\int_{-\infty}^{\infty} \frac{f(x,0)}{x-x_0}\, dx$$



This result comes from considering $\int_\gamma \frac{f(z)\,dz}{z-x_0}$

The integral is zero since there are no singularities inside γ.

$\int_{\gamma_R} \frac{f}{z-x_0} \to 0$ since $\left|\frac{f}{z-x_0}\right|$ goes to zero faster than R.

We are left with

$$P\int_{-\infty}^{\infty} \frac{f(x,0)\,dx}{x-x_0} - i\pi\, f(x_0,0) = 0$$

Cauchy P.V.

from which the relation follows.