

UCSD Physics Departmental  
Written Examination

Spring 1991

Department of Physics  
University of California, San Diego  
La Jolla, California 92093-0319  
Written Departmental Examination - Spring, 1991

PART I

**Instructions**

Each problem is worth 10 points. This part has 9 problems.

**Some Information** (for both PART I and PART II)

Hydrogen radial wavefunctions:

$$R_{1s} = \frac{2}{a_0^{3/2}} e^{-r/a_0}$$
$$R_{2s} = \frac{1}{(2a_0)^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$$
$$R_{2p} = \frac{1}{(2a_0)^{3/2}} \frac{r}{a_0\sqrt{3}} e^{-r/2a_0}$$

Stirling's approximation

$$n! = (2\pi n)^{1/2} n^n e^{-n}$$

Gravitational constant

$$G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$$

Mass of earth

$$M = 5.98 \times 10^{24} \text{ kg}$$

Radius of earth

$$R = 6.37 \times 10^6 \text{ m}$$

Mass of moon

$$m = 7.4 \times 10^{22} \text{ kg}$$

### Problem 1

A particle in three dimensions is in a state with wavefunction

$$\psi(r) = C (3iY_{22} - Y_{20} + 2Y_{21}) R(r)$$

where  $R(r)$  is some arbitrary radial wavefunction normalized such that

$$\int_0^\infty |R(r)|^2 r^2 dr = 1,$$

and the  $Y_{lm}$  are spherical harmonics.

- Find a value of  $C$  that will normalize the wavefunction.
- If a measurement of  $L_z$  is made, what are the possible measured values and what are the probabilities for each?
- Find the expectation value of  $L_x$  in the above state.

### Problem 2

A particle of mass  $m$  starts from rest at the edge of a hemispherical bowl which is held fixed. The coefficient of friction between the particle and bowl is  $\mu$ .

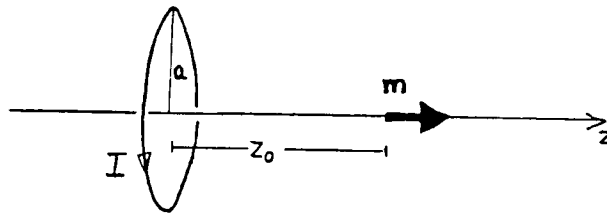
- Find the velocity of the particle when it reaches the bottom of the bowl under the action of gravity.
- What condition determines the maximum value of  $\mu$  for which the particle reaches the bottom?



### Problem 3

A particle of mass  $M$  and magnetic dipole moment  $\mathbf{m}$  is placed on the axis of a circular current loop of radius  $a$  and current  $I$  (which is kept fixed), at a distance  $z_0$  from the center of the loop.  $\mathbf{m}$  is aligned in the direction of the loop field. ( $z_0$  is not necessarily much greater or smaller than  $a$ .)

- What is the force of attraction between the loop and  $\mathbf{m}$ ?
- When  $\mathbf{m}$  is released, it moves toward the center of the loop. What is its kinetic energy when it arrives there? (Assume that  $\mathbf{m}$  is constrained to the  $z$  axis.)
- If the particle is originally placed at the center of the loop, what is the frequency of small oscillations about this position for motion along the  $z$  axis?



### Problem 4

In an experiment in which J. Perrin determined the value of Avogadro's number  $N_A$ , a suspension of globules of gamboge ( $\rho = 1.254 \text{ g/cm}^3$ ) in water was suspended at a temperature of  $20^\circ\text{C}$ . The radius of the particles was  $0.212 \mu\text{m}$ . When the field of view of the microscope was raised by a distance of  $30 \mu\text{m}$  the number of globules seen was changed by 2.1 times. Find the value for  $N_A$ . ( $R = 8.31 \text{ J/mol K}$ )

### Problem 5

- (a) What is the value of the escape velocity from the moon's surface?
- (b) Why is the sky blue and the sunset red?
- (c) What is the range of wavelengths of visible light?
- (d) Write down the Lorentz gauge condition.
- (e) What is the "second virial coefficient"?
- (f) Between H, He and Li, which has the lowest and which the highest ionization potential? Why?
- (g) What are typical Debye temperatures in solids?
- (h) What is the mass of the proton?

### Problem 6

For purposes of electrostatics, a protein may be modeled as a medium with a low dielectric constant, say  $\epsilon_p/\epsilon_0 \approx 4$ . A charge  $Q$  is placed in a protein which is in aqueous solution (the dielectric constant of water is  $\epsilon_w/\epsilon_0 = 80$ ). The electrostatic energy, compared with that of the same charge in an infinite medium with the same dielectric constant as the protein, is called the "solvation energy".

- (a) Suppose that the protein is a sphere of radius  $a$  and the charge is at the center of the sphere. What is the solvation energy? 5
- (b) Real proteins are not spherical. Describe briefly how you would compute the solvation energy for a charge in a real protein of known shape. 2
- (c) In general, how will the solvation energy change if  $\epsilon_p/\epsilon_0 \approx 2$ ? (Give an approximate answer.) 3

### Problem 7

Show using a simple model with Newtonian gravity that the force of the sun on the earth's tides is about 1/2 that of the moon. In what phase of the lunar cycle (*e.g.*, new, 1st quarter, full, last quarter) would you expect very high and low tides? Show the relative positions of earth, sun and moon in a diagram.

The earth loses angular momentum as a result of tidal friction. What result does this have on the motions of the earth and moon?

mass of moon	=	$7.4 \times 10^{25}$ g
mass of sun	=	$2 \times 10^{33}$ g
av distance earth to moon	=	$3.8 \times 10^{10}$ cm
av distance earth to sun	=	$1.5 \times 10^{13}$ cm

### Problem 8

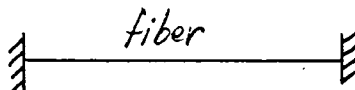
A beam of particles with energy  $E > 0$  coming from  $-\infty$  is incident on a potential step in one dimension, with  $V(x) = 0$  for  $x < 0$ , and  $V(x) = -V_0$  for  $x > 0$ .  $V_0$  is a positive real number.

- Find the general solution to the Schrödinger equation for this problem in the regions  $x > 0$  and  $x < 0$ .
- Determine the coefficients needed to satisfy the boundary conditions.
- Calculate the reflection and transmission probabilities, and determine the total probability for reflection or transmission.

### Problem 9

A thin fiber of length  $L$  is stretched between two supports. The speed of propagation of transverse waves on the fiber is  $c$  for both polarizations.

- (a) What is the contribution of these modes to the heat capacity of the fiber at low temperatures, assuming  $hc/L \ll kT$ ? Leave any integral in dimensionless form to display the temperature dependence explicitly.
- (b) What is the heat capacity for  $hc/L \gg kT$ ?



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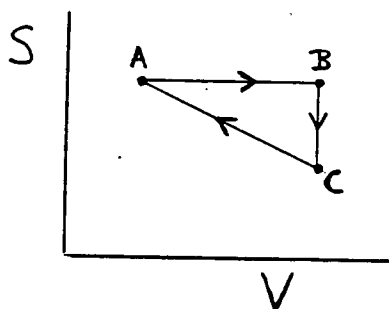
Mass of moon

$$m = 7.4 \times 10^{25} \text{ g}$$



### Problem 10

A peculiar thermodynamic engine operates according to the following diagram:



All lines in the above thermodynamic path are straight lines. AB is horizontal and BC is vertical.

- Sketch what this diagram would look like in the  $p$ - $V$  plane.
- Suppose the gas used in the engine is an ideal gas with specific heat  $C_V = \text{const}$  per mole. Calculate the equation for the segment AC in the  $p$ - $V$  plane. [This means obtain a function  $p(V)$  which describes the segment AC. Your answer may involve constants like  $S_A, S_C, V_A, V_C$  and  $n$  (number of moles).]

### Problem 11

Consider the scattering of a spin-1/2 particle via the interaction

$$V = \lambda [V_Y(x) \mathbf{s} \cdot \mathbf{p} + \mathbf{s} \cdot \mathbf{p} V_Y(x)]$$

where

$$V_Y(x) = V_0 \frac{e^{-\mu r}}{\mu r}$$

is a Yukawa potential.

- What is the Born approximation for the spin-flip scattering amplitude, *i.e.* for an incident beam polarized parallel to the incident momentum direction to scatter to an outgoing beam with spin opposite to the outgoing momentum direction?
- Show that the scattering process violates parity, but preserves time-reversal invariance.

### Problem 12

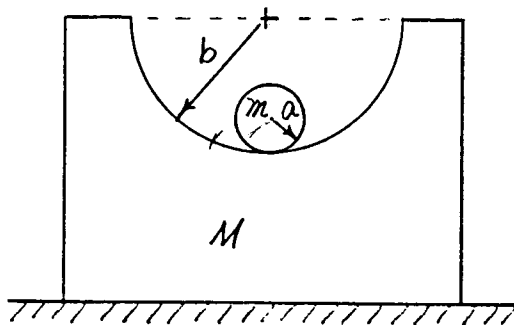
The  $2S_{1/2}$  state of hydrogen cannot decay to the ground state via a one photon process.

- (a) Prove the above statement.
- (b) Now turn on a weak electric field of magnitude  $\mathcal{E}$ . If the splitting between the  $2S_{1/2}$  and  $2P_{1/2}$  states is  $E_L$ , and the lifetime for the one photon decay of  $2P_{1/2}$  is  $\tau_P$ , find the decay rate  $\tau_S$  of the  $2S_{1/2}$  state. The answer should be given in terms of  $\tau_P$ ,  $\mathcal{E}$ ,  $E_L$ , and the Bohr radius  $a_0$ .

### Problem 13

A solid cylinder of radius  $a$ , length  $l$ , and mass  $m$ , rolls inside a cylindrical groove of radius  $b > a$  in a block of mass  $M$ , as shown in the figure. The block rests on a frictionless horizontal surface. The moment of inertia of the solid cylinder about the cylinder axis is  $I = ma^2/2$ .

- (a) Write the Lagrangian for the system using a clear diagram to label your generalized coordinates.
- (b) Using the appropriate small angle expansion, find the characteristic frequencies.
- (c) Discuss the normal modes.



### Problem 14

Consider the scattering of an electromagnetic wave by a perfectly conducting sphere in the quasi-static limit (*i.e.* wavelength large compared to the radius of the sphere). The incident wave propagates in the positive  $z$  direction, and the wave is plane polarized with the electric field in the  $x$  direction.

- (a) Find the electric dipole moment induced in the sphere by a uniform static electric field and the magnetic dipole moment induced in the (perfectly diamagnetic) sphere by a uniform static magnetic field.
- (b) Find the ratio of the scattered intensity in the  $x$ ,  $y$ , and backward  $z$  directions to that in the forward  $z$  direction.

### Problem 15

A function  $y(x)$  obeys the differential equation.

$$y'''' - x^3y = 0$$

- (a) What is the general solution for  $y$  valid near  $x = 0$  which has three independent constants of integration?
- (b) Almost all solutions of the differential equation will behave at large  $x$  as  $y = Cx^\alpha \exp(\beta x^\gamma)$ . Find  $\alpha$ ,  $\beta$ ,  $\gamma$ . Hint: Use the WKB method.
- (c) At  $x = 0$ ,  $y = 1$ ,  $y' = y'' = 0$ . Find a power series expression for  $y(x)$  which is valid for all  $x$ .
- (d) Using asymptotic methods evaluate the series found in part (c) for large  $x$ . Show that the answer reduces to that found in part (b) and determine the constant  $C$ . Hint: Use Stirling's approximation.

### Problem 16

- (a) Explain how the magnetic behavior of a "superconductor" differs from that of a "perfect conductor".
- (b) Give a simple derivation of Bragg's law for x-ray diffraction from a crystal. Explain how you would use your result to determine the spacings between crystal planes.
- (c) Describe how you would produce an electron beam with velocity  $\vec{v}$  and use its interaction with uniform electric and magnetic fields  $\vec{E}$  and  $\vec{B}$ , respectively, to determine the charge-to-mass ratio  $e/m$  of the electron. Give expressions for the deflection of the electron beam in the  $\vec{E}$  and  $\vec{B}$  fields.

### Problem 17

A free electron Fermi gas of  $N$  particles is contained in a thermally insulated box of volume  $V$  at zero temperature. Its Fermi energy is  $\epsilon_F$ .

- (a) The system is allowed to expand quasi-statically until its volume reaches  $V' = 1.1 V$ . Find the work done by the system on the environment.
- (b) Instead, the system undergoes a free expansion from  $V$  to  $V' = 1.1 V$ . Find (approximately) the change in its temperature and give an order-of-magnitude estimate of the error in your result.

Note: Give your answers in terms of the given quantities  $\epsilon_F$ ,  $V$ , and  $N$ .

SP9T

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 10

Please insert on page

the Problem No. 1 and your Identification No. 29

(a)  $\int |\psi|^2 d^3r = 1 = \int d\Omega c^2 (-3i Y_{32}^* - Y_{20}^* + 2 Y_{21}^*) (3i Y_{22} - Y_{20} + 2 Y_{21})$

$\therefore c^2 (9 + 1 + 4) = 1$

$\therefore c = \frac{1}{\sqrt{14}}$

(b) Let P stand for probability

then:  $L_z = 2\hbar \quad P = \frac{9}{14}$

$L_z = \hbar \quad P = \frac{4}{14} = \frac{2}{7}$

$L_z = 0 \quad P = \frac{1}{14}$

(c)  $L_x = \frac{1}{2}(L_+ + L_-)$

$\therefore L_x (3i Y_{22} - Y_{20} + 2 Y_{21})$

$= \frac{1}{2} (L_+ + L_-) (3i Y_{22} - Y_{20} + 2 Y_{21})$

$= \frac{1}{2} \left[ 3i \sqrt{(2+2)(2-2+1)} Y_{21} - (\sqrt{(2-0)(2+0+1)} Y_{21} + \sqrt{(2+0)(2-0+1)} Y_{2,-1}) \right.$

$\left. + 2 (\sqrt{(2-1)(2+1+1)} Y_{22} + \sqrt{(2+1)(2-1+1)} Y_{20}) \right] \hbar$

$= \left[ 2 Y_{22} + (3i - \frac{1}{2}\sqrt{6}) Y_{21} + \sqrt{6} Y_{20} - \frac{\sqrt{6}}{2} Y_{2,-1} \right] \hbar$

$\therefore \langle L_x \rangle = \frac{1}{14} \left[ 2 \times (-3i) + (-1) (\frac{1}{2}\sqrt{6}) + 2 (3i - \frac{1}{2}\sqrt{6}) \right] \hbar = -\frac{\sqrt{6}}{7} \hbar$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 5

Please insert on page

the Problem No. 20 and your Identification No. 29

(a)



$\theta$  as shown. +1

$$\text{Then } N = mg \sin \theta + \frac{mV^2}{R}$$

$$\text{The friction } f = -\mu \left( mg \sin \theta + \frac{mV^2}{R} \right)$$

~~The work done by~~

$$\text{The negative work by friction } W = \int f ds + 2$$

$$= - \int_0^{\pi} \mu mg \sin \theta R d\theta$$

$$= -\mu mg R \int_0^{\pi} \sin \theta d\theta = -\mu mg R$$

$$\therefore \frac{1}{2} m v_f^2 + \mu mg R = mg R$$

$$v_f = \sqrt{2gR(1-\mu)}$$

(b) from (a)  $\mu = 1$  +1

the condition is  $v_f = \sqrt{2gR(1-\mu)} = 0$

yes, but leaving out radial acceleration of circular motion made this problem too easy!

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score =

10

Please insert on page

the Problem No. 3 and your Identification No. 29

(a) 
$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint \frac{Id\vec{l} \times \vec{R}}{R^3} = \frac{\mu_0 I}{C} \cdot \frac{2\pi a}{R^2} \cdot \frac{a}{R} \hat{z} = \frac{2\pi I}{C} \frac{a^2}{R^3} \hat{z}$$

$$\therefore \vec{B} = \frac{2\pi I a^2}{C(a^2 + z_0^2)^{3/2}} \hat{z}$$



The force on  $\vec{m}$  is 
$$\vec{F} = \frac{\partial}{\partial z_0} (\vec{B} \cdot \vec{m}) = \frac{2\pi I a^2 m}{C} \frac{\partial}{\partial z_0} \frac{1}{(a^2 + z_0^2)^{3/2}} \hat{z}$$

$$= \frac{2\pi I a^2 m}{C} \cdot \left(-\frac{3}{2}\right) \frac{2z_0}{(a^2 + z_0^2)^{5/2}} \hat{z}$$

$$= \left(-\frac{3}{2}\right) \frac{6\pi I m a^2 z_0}{C(a^2 + z_0^2)^{5/2}} \hat{z}$$

\*  $\vec{F} = \nabla \cdot (\vec{m} \cdot \vec{B})$ , by symmetry consideration,  $\vec{F}$  only has  $\hat{z}$  component  $\therefore \vec{F} = \hat{z} \frac{\partial}{\partial z_0} (\vec{m} \cdot \vec{B})$

(b) The kinetic energy  $T = -\Delta U$   $U = -(\vec{m} \cdot \vec{B})$

$$= \Delta (\vec{m} \cdot \vec{B})$$

$$= \vec{m} \cdot \vec{B} \Big|_{z_0=0} - \vec{m} \cdot \vec{B} \Big|_{z_0}$$

$$= \frac{2\pi I a^2 m}{C} \left[ \frac{1}{a^3} - \frac{1}{(a^2 + z_0^2)^{3/2}} \right]$$

(c)  $U = -(\vec{m} \cdot \vec{B})$

let  $U_0$  be the potential energy at  $z_0 = 0$ .

for  $z_0 \ll a$

$$U - U_0 = \frac{2\pi I a^2 m}{C a^3} \left[ 1 - \frac{1}{\left[1 + \left(\frac{z_0}{a}\right)^2\right]^{3/2}} \right] = \frac{2\pi I m}{C a} \cdot \frac{3}{2} \left(\frac{z_0}{a}\right)^2$$

$$\therefore U - U_0 = \frac{3\pi I m}{C a^3} z_0^2 = \frac{1}{2} M \omega^2 z_0^2$$

$$\therefore \omega = \sqrt{\frac{6\pi I m}{M C a^3}} \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{6\pi I m}{M C a^3}}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 9

Please insert on page

the Problem No. 4

and your Identification No. 29

Solution:

$$n = n_0 e^{-\frac{mg}{RT} z}$$

← correct Sm

$$\therefore e^{\frac{mg}{RT} (z_2 - z_1)} = \frac{n_1}{n_2}$$

Boyle's Law

$$\therefore \frac{mg}{RT} (z_2 - z_1) = \ln \frac{n_1}{n_2}$$

$$k = \ln \frac{n_2}{n_1} \cdot \frac{mg}{T} (z_2 - z_1)$$

$$m = \rho \cdot \frac{4}{3} \pi r^3$$

$$\therefore k = \left( \ln \frac{n_2}{n_1} \right) \cdot \frac{g(z_2 - z_1)}{T} \times \frac{4}{3} \pi r^3 \rho$$

$$R = N_A k$$

$$\therefore N_A = \frac{R}{k} = \frac{R}{\ln \left( \frac{n_2}{n_1} \right) \cdot \frac{g(z_2 - z_1)}{T} \times \frac{4}{3} \pi r^3 \rho}$$

8.31

$$= \frac{8.31}{(2.1) \times \frac{9.8 \times 30 \times 10^{-6}}{20 + 273} \times \frac{4}{3} \pi \times (0.212 \times 10^{-6})^3 \times 1.254 \times 10^3}$$

$$= \frac{8.31}{3.73 \times 10^{-25}} = 2.23 \times 10^{25} \text{ (1/mol)}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 7

Please insert on page

the Problem No. 5-1

and your Identification No. 29

(a)  $\frac{1}{2} m v^2 = \frac{m G M_m}{R_m}$

$\therefore v = \sqrt{\frac{2 G M_m}{R_m}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{1.74 \times 10^6}} = 2.38 \times 10^3 \text{ (m/s)}$

~~suppose moon and earth have same density then  $\frac{M_m}{M_e} = \frac{R_m^3}{R_e^3} \Rightarrow R_m = R_e \sqrt[3]{\frac{M_m}{M_e}}$~~

~~$\therefore v = \sqrt{\frac{2 G M_m \cdot (M_e)^{1/3}}{(M_m) R_e}}$~~

~~$= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.4 \times 10^{22} \cdot (5.98 \times 10^{24})^{1/3}}{1.4 \times 10^6 \times 6.37 \times 10^6}}$~~   
 ~~$\approx 2.59 \times 10^3 \text{ (m/s)}$~~

(b) The scattering cross section of ~~that~~ Rayleigh scattering is proportional to  $\frac{1}{\lambda^4}$ , for short wave length light (blue) gets more scattered. And the direct light consists more of long wave length, which is more red.

(c)  $3500 \text{ \AA} \sim 7500 \text{ \AA}$  ✓

(d)  $\nabla \cdot \vec{A} + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0$  ✓

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

3

  
 Score =

Please insert on page

the Problem No. 6

and your Identification No. 29

(a)

$$\vec{E} = \frac{1}{\epsilon} \frac{Q}{r^2} \hat{r} \quad \text{energy } U = \frac{1}{8\pi} \vec{E} \cdot \vec{D} = \frac{\epsilon}{8\pi} E^2$$

$$\begin{aligned} \therefore \text{The solvation energy} &= \frac{1}{8\pi} \int_a^\infty 4\pi r^2 dr \left[ \epsilon_p \left( \frac{Q}{\epsilon_p r^2} \right)^2 - \epsilon_w \left( \frac{Q}{\epsilon_w r^2} \right)^2 \right] \\ &= \frac{Q^2}{2} \left( \frac{1}{\epsilon_p} - \frac{1}{\epsilon_w} \right) \int_a^\infty \frac{dr}{r^2} \\ &= \frac{Q^2}{2} \left( \frac{1}{4} - \frac{1}{80} \right) \frac{1}{a} \\ &= \frac{19}{160} \frac{Q^2}{a} \end{aligned}$$

(b)

(c) If  $\epsilon_p/\epsilon_w \approx 2$ , the solvation energy will be doubled. 3

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 1

Please insert on page

the Problem No. 7-1

and your Identification No. 29

The simple model for tides is : ~~the~~ The centrifugal forces of different points on the earth are different.

let  $m$  be the mass of the earth,  $r_c$  is distance to the center of the earth.

Then: 
$$\frac{GmM}{r_c^2} = mr_c \omega^2$$

$$\therefore m\omega^2 = \frac{GmM}{r_c^3}$$

NO.  
tidal & differential force between center & surface of earth.

let  $R_e$  be the radius of earth.

The tidal force  $f_t = mR\omega^2 = \frac{GmM R}{r_c^3} = GMR \frac{M}{r_c^3}$

$$\therefore \frac{f_{t \text{ moon}}}{f_{t \text{ sun}}} = \frac{M_m / r_{m-e}^3}{M_s / r_{s-e}^3} = \frac{M_m \cdot r_{s-e}^3}{M_s \cdot r_{m-e}^3}$$

$$= \frac{7.4 \times 10^{25} \times (1.5 \times 10^{13})^3}{2 \times 10^{33} \times (3.8 \times 10^{10})^3}$$

$$= \frac{74 \times 1.5^3}{2 \times 3.8^3} = 2.3$$

right answer wrong method.

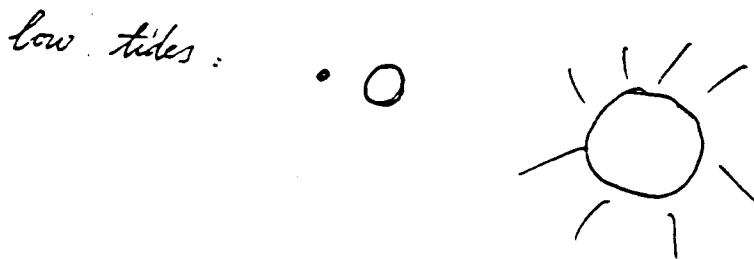
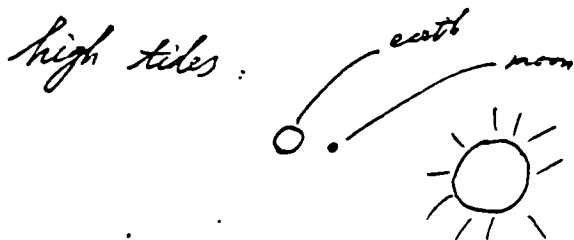
PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 4

Please insert on page

the Problem No. 7-2 and your Identification No. 29

We would expect high <sup>low</sup> tides at full moon  
low tides <sup>high</sup> at new moon



The earth loses angular momentum as a result of tidal friction, this will result a radial ~~motion~~ <sup>motion</sup> between the earth and the moon, so they get ~~more close~~ <sup>earth-moon distance increases</sup> to each other. Actually, there will be a small ~~oscillation~~ <sup>oscillation</sup> radial oscillation about the equilibrium position.

We see from  $\frac{GMm}{r_c^2} = m r_c \omega^2 = m r_c \frac{J^2}{m^2 r_c^4} = \frac{J^2}{m r_c^3} \Rightarrow Gm^2 M r_c = J^2$

Note: If you use additional sheets for this problem, number the pages and staple them together.

decrease of  $J$  cause decrease of  $r_c$ , but eventually this will result an oscillation.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

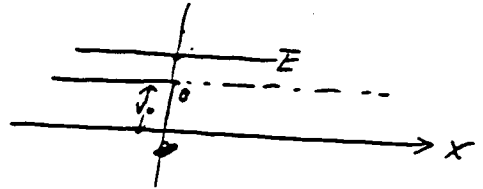
Score = 10

Please insert on page

the Problem No. 8 and your Identification No. 29

(a)

$$\psi(x) = \begin{cases} e^{ikx} + Ae^{-ikx} & x < 0 \\ Be^{ik'x} & x > 0 \end{cases}$$



$$k = \sqrt{\frac{2mE}{\hbar^2}}, \quad k' = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$$

(b) At  $x=0$ :

$$\begin{cases} 1 + A = B \\ ik(1 - A) = ik'B \Rightarrow 1 - A = \frac{k'}{k} B \end{cases}$$

$$\therefore B = \frac{2k}{k+k'}$$

$$A = \frac{k-k'}{k+k'}$$

$$\therefore \psi(x) = \begin{cases} e^{ikx} + \frac{k-k'}{k+k'} e^{-ikx} & x < 0 \\ \frac{2k}{k+k'} e^{ik'x} & x > 0 \end{cases}$$

(c) since  $\vec{j} = \text{Re} \langle \psi | -\frac{i\hbar}{m} \nabla | \psi \rangle$

$$\text{The reflection probability } R = \left| \frac{k-k'}{k+k'} \right|^2 = \frac{(k-k')^2}{(k+k')^2}$$

$$\text{The transmission probability } T = \frac{k'}{k} \cdot \left( \frac{2k}{k+k'} \right)^2 = \frac{4kk'}{(k+k')^2}$$

$$\text{Total: } R + T = 1$$

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Solution: The modes are  $k_n = \frac{n\pi}{L}$ ,  $n=1, 2, 3, \dots$

They satisfy Bose statistics

~~They satisfy Bose statistics~~

(a)  $\omega = kc$

$\therefore E = \sum_n \frac{\hbar \omega_n}{e^{\hbar \omega_n / kT} - 1} \cdot 2$  ← polarization

for  $\frac{\hbar c}{L} \ll kT$

$E \approx \frac{2L}{\pi} \int_0^\infty dk \frac{\hbar \omega}{e^{\hbar \omega / kT} - 1}$

$= \frac{2L}{\pi c} \int_0^\infty d\omega \frac{\hbar \omega}{e^{\hbar \omega / kT} - 1}$

$= \frac{2L}{\pi c} \cdot \hbar \left(\frac{kT}{\hbar}\right)^2 \int_0^\infty \frac{x dx}{e^x - 1}$

$= \frac{2L}{\pi \hbar c} (kT)^2 \int_0^\infty \frac{x dx}{e^x - 1}$

$\therefore C_v = \frac{\partial E}{\partial T} = \frac{4L k^2}{\pi \hbar c} T$  ✓

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(b)

$$E = \sum_n \frac{2 \hbar \omega_n}{e^{\hbar \omega_n / kT} - 1}$$

$$\omega_n = k_n c = \frac{\pi c}{L} \cdot n$$

$$= 2 \cdot \sum_n \frac{\frac{\pi \hbar c}{L} \cdot n}{e^{\frac{(\pi \hbar c \cdot n) / kT}{L}} - 1}$$

if  $\frac{\hbar c}{L} \gg kT$

$$E \approx 2 \sum_{n=1}^{\infty} \frac{n \pi \hbar c}{L} e^{-\frac{(n \pi \hbar c) / kT}{L}}$$

$$\approx 2 \cdot \frac{\pi \hbar c}{L} e^{-\pi \hbar c / L kT}$$

(n=1)

$$\therefore E = \frac{2 \pi \hbar c}{L} e^{-\pi \hbar c / L kT}$$

$$C_v = \frac{\partial E}{\partial T} = \frac{2 \pi \hbar c}{L} \cdot \frac{\pi \hbar c}{L k} \cdot \frac{1}{T^2} e^{-\frac{\pi \hbar c}{L kT}}$$

$$= \left(\frac{\pi \hbar c}{L}\right)^2 \frac{2}{kT^2} e^{-\frac{\pi \hbar c}{L kT}}$$

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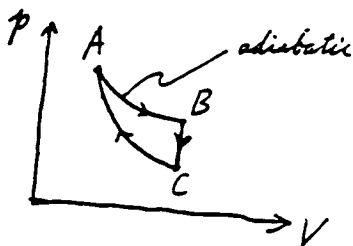
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(a)



or  
(possible)



but system is an engine  $\Rightarrow \Delta W > 0$   
so your diagram is right

(b)

$$dE = Tds - PdV \checkmark$$

$$dE = nC_v dT \checkmark$$

$$P = \frac{nRT}{V} \checkmark$$

$$\therefore nC_v dT = Tds - \frac{nRT}{V} dV$$

$$\therefore ds = \frac{nC_v}{T} dT + \frac{nR}{V} dV \checkmark$$

$$\therefore S(V, T) = S(V_0, T_0) + nC_v \ln \frac{T}{T_0} + nR \ln \frac{V}{V_0} \checkmark \quad (1)$$

along AC:  $S = \frac{V_C S_A - V_A S_C}{V_C - V_A} - \frac{S_A - S_C}{V_C - V_A} V \quad (2)$

from (1), (2):

$$\frac{V_C S_A - V_A S_C}{V_C - V_A} - \frac{S_A - S_C}{V_C - V_A} V = S_A + nC_v \ln \frac{T}{T_A} + nR \ln \frac{V}{V_A}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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This can be solved for  $T = T(V)$  ✓

$$nC_V \ln \frac{T}{T_A} = \frac{V_C S_A - V_A S_C}{V_C - V_A} - \frac{S_A - S_C}{V_C - V_A} V - S_A - nR \ln \frac{V}{V_A}$$

~~$T = T_A \exp \left[ \frac{1}{nC_V} \left( \frac{V_C S_A - V_A S_C}{V_C - V_A} - \frac{S_A - S_C}{V_C - V_A} V - S_A - nR \ln \frac{V}{V_A} \right) \right]$~~

$$T = T_A \exp \left[ \frac{1}{nC_V} \left( \frac{V_C S_A - V_A S_C}{V_C - V_A} - \frac{S_A - S_C}{V_C - V_A} V - S_A - nR \ln \frac{V}{V_A} \right) \right]$$

$$P = \frac{nRT}{V}$$

$$= \frac{nRT_A}{V} \cdot \left( \frac{V_A}{V} \right)^{\frac{R}{C_V}} \exp \left[ \frac{1}{nC_V} \left( \frac{V_C S_A - V_A S_C}{V_C - V_A} - \frac{S_A - S_C}{V_C - V_A} V - S_A \right) \right]$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 2

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(a)

Incident :  $\psi = e^{ikz} \alpha = e^{ikz} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

outgoing :  $\psi = \frac{f(\theta)}{r} e^{ikr} \chi$

+0,  $\theta$  dir.  $\chi = \begin{pmatrix} \sin \frac{\theta}{2} e^{i\phi} \\ \cos \frac{\theta}{2} \end{pmatrix} e^{i\phi/2}$

-0,  $\theta$  dir.  $\chi = \begin{pmatrix} \cos \frac{\theta}{2} e^{i\phi} \\ -\sin \frac{\theta}{2} \end{pmatrix} e^{-i\phi/2}$

$\therefore$  now outgoing  $\psi = \frac{f(\theta)}{r} e^{ikr} \begin{pmatrix} \cos \frac{\theta}{2} \\ -\sin \frac{\theta}{2} \end{pmatrix}$

$(1, 0) S_x \begin{pmatrix} \cos \frac{\theta}{2} \\ -\sin \frac{\theta}{2} \end{pmatrix} = -\sin \frac{\theta}{2} e^{-i\phi/2}$

$(1, 0) S_y \begin{pmatrix} \cos \frac{\theta}{2} \\ -\sin \frac{\theta}{2} \end{pmatrix} = -i \sin \frac{\theta}{2} e^{i\phi/2}$

$(1, 0) S_z \begin{pmatrix} \cos \frac{\theta}{2} \\ -\sin \frac{\theta}{2} \end{pmatrix} = \cos \frac{\theta}{2} e^{i\phi/2}$

$\therefore f_{\pm}(\theta, \phi) = -\frac{M}{2\pi\hbar^2} \int d^3r e^{-ikr} [V_r(\vec{r}) (-\sin \frac{\theta}{2} p_x - i \sin \frac{\theta}{2} p_y + \cos \frac{\theta}{2} p_z) e^{-i\phi/2} + i (-\sin \frac{\theta}{2} p_x - i \sin \frac{\theta}{2} p_y + \cos \frac{\theta}{2} p_z) V_r(\vec{r})] e^{i\phi/2}$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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~~for spin unpolarized.~~

~~can get  $f(\theta, \varphi)$  by  $\theta \rightarrow \pi$ .~~

$$\therefore f(\theta, \varphi) = -\frac{\mu}{2\pi\hbar^2} \int d^3r e^{-ikr} [V(r)(-\frac{\epsilon}{2}e^{-i\frac{\varphi}{2}}p_x - i\frac{\epsilon}{2}e^{-i\frac{\varphi}{2}}p_y + a e^{i\frac{\varphi}{2}}p_z) + (-\frac{\epsilon}{2}e^{-i\frac{\varphi}{2}}p_x - i\frac{\epsilon}{2}e^{-i\frac{\varphi}{2}}p_y + a e^{i\frac{\varphi}{2}}p_z)V(r)]e^{i\varphi}$$

(b)

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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(a) The ground state is  $1S_{1/2}$ , it is spherically symmetric in orbital space same is true for  $2S_{1/2}$ .  $\therefore \langle 1S_{1/2} | \vec{r} | 2S_{1/2} \rangle = 0$  by symmetry considerations. Thus  $2S_{1/2} \rightarrow 1S_{1/2}$  is not allowed.  
+3

(b)  $2S_{1/2} \rightarrow 2P_{1/2}$

$$\frac{1}{\tau} \propto \omega^3 \propto (kE)^3$$

$$\therefore \frac{\frac{1}{\tau_p}}{\frac{1}{\tau_s}} = \frac{\left(\frac{e^2}{4a_0}\right)^3}{2 \cdot E_L^3} = \frac{1}{2} \left(\frac{e^2}{4a_0 E_L}\right)^3$$

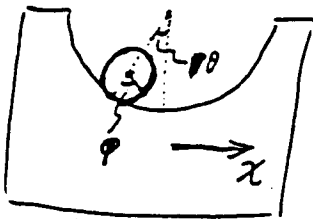
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(a)



use  $\theta$  and  $x$  as generalized coordinates  
there is a ~~sub~~ relation between  $\theta$  and  $\varphi$

~~$b\theta = a\varphi$~~   $b\theta = a\varphi$   $\varphi = \frac{(b-a)\theta}{a}$   
 $a\dot{\varphi} = (b-a)\dot{\theta}$

$V = -mg(b-a)\cos\theta$

$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}I\dot{\varphi}^2 + \frac{1}{2}m[(b-a)\dot{\theta}\sin\theta]^2$   
 $+ \frac{1}{2}m[(b-a)\dot{\theta}\cos\theta - \dot{x}]^2$  OK.  
 $= \frac{1}{2}M\dot{x}^2 + \frac{1}{4}mb^2\dot{\theta}^2 + \frac{1}{2}m(b-a)^2\dot{\theta}^2 + \frac{1}{2}m\dot{x}^2 - m(b-a)\dot{\theta}\dot{x}\cos\theta$

$\therefore L = T - V = \frac{1}{2}M\dot{x}^2 + \frac{1}{4}mb^2\dot{\theta}^2 + \frac{1}{2}m(b-a)^2\dot{\theta}^2 + \frac{1}{2}m\dot{x}^2$   
 $- m(b-a)\dot{\theta}\dot{x}\cos\theta + mg(b-a)\cos\theta$   
 $= \frac{1}{2}(M+m)\dot{x}^2 - m(b-a)\dot{\theta}\dot{x}\cos\theta + \frac{1}{2}m[(b-a)^2 + \frac{1}{2}b^2]\dot{\theta}^2$   
 $+ mg(b-a)\cos\theta$  OK.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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$$\frac{\partial L}{\partial \dot{x}} = (M+m)\dot{x} - m(b-a)\dot{\theta}\cos\theta = p_x = \text{const.}$$

$$\therefore \dot{x} = \frac{m(b-a)\dot{\theta}\cos\theta}{M+m} + \frac{p_x}{M+m}$$

substitute back to L:

$$L = \frac{1}{2}(M+m) \left[ \frac{m(b-a)\dot{\theta}\cos\theta}{M+m} + \frac{p_x}{M+m} \right]^2 - m(b-a)\dot{\theta}\cos\theta \left[ \frac{m(b-a)\dot{\theta}\cos\theta}{M+m} + \frac{p_x}{M+m} \right] + \frac{1}{2}m[(b-a)^2 + \frac{1}{2}b^2]\dot{\theta}^2 + mg(b-a)\cos\theta$$

$$= -\frac{1}{2} \frac{m^2(b-a)^2\dot{\theta}^2\cos\theta}{M+m} + \frac{1}{2} \frac{p_x^2}{M+m} + \frac{1}{2}m[(b-a)^2 + \frac{1}{2}b^2]\dot{\theta}^2 + mg(b-a)\cos\theta$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 5

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(b) To second order of  $\theta$ : (and  $\dot{\theta}$ )

$$L = \left\{ \frac{1}{2} m [(b-a)^2 + \frac{1}{2} b^2] - \frac{1}{2} \frac{m^2 (b-a)^2}{M+m} \right\} \dot{\theta}^2 + \frac{1}{2} \frac{p_x^2}{M+m}$$

$$+ mg(b-a) \left(1 - \frac{1}{2} \theta^2\right)$$

$$\therefore \omega^2 = \frac{mg(b-a)}{\frac{1}{2} m [(b-a)^2 + \frac{1}{2} b^2] - \frac{1}{2} \frac{m^2 (b-a)^2}{M+m}}$$

$$= \frac{2g(b-a)(M+m)}{M(b-a)^2 + \frac{1}{2}(M+m)b^2}$$

$$\omega = \sqrt{\frac{2g(b-a)(M+m)}{M(b-a)^2 + \frac{1}{2}(M+m)b^2}}$$

what about  
eqn. of  
motion?

$$(c) \quad \dot{x} = \frac{m(b-a)\dot{\theta} \cos \theta}{M+m} + \frac{p_x}{M+m}$$

in a frame where  $p_x = 0$   
and for small oscillation  $\cos \theta \sim 1$

$$\therefore \dot{x} = \frac{m(b-a)}{M+m} \dot{\theta}$$

$$\text{and } x = \frac{m(b-a)}{M+m} \theta$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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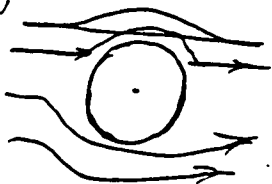
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(a)



For uniform static electric field

$$r < R \quad \varphi_1 = \varphi_0$$

$$r > R \quad \varphi_2 = \varphi_0' + (-Er + \frac{A}{r^2}) \cos \theta$$

$$\text{at } r = R \quad \varphi_1 = \varphi_2$$

$$\therefore \varphi_0 = \varphi_0'$$

$$ER = \frac{A}{R^2} \Rightarrow A = ER^3$$

$$\therefore \text{outside } \varphi = \varphi_0 - E(r - \frac{R^3}{r^2}) \cos \theta$$

the dipole potential is  $\frac{ER^3}{r^2} \cos \theta = \frac{\vec{p} \cdot \vec{r}}{r^3}$

$$\therefore \vec{p} = R^3 \vec{E}$$

Similarly, since there is no current density, we can use the concept of magnetic charge, and get the result by analog

$$\vec{m} = R^3 \vec{B} \times$$

$B_r = 0$  is exact boundary condition for diamagnet



PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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(b) Now  $\vec{E}$  is in  $\hat{x}$  direction  
 $\vec{B}$  is in  $\hat{y}$  direction

$\vec{p}$  in same direction as  $\vec{E}$ , that is  $\hat{x}$ .

for ~~the~~ electric dipole scattering (radiation), (This is the highest order contribute)

~~scattered field  $\vec{E}_s$  is in  $\hat{x}$  direction that is  $\hat{x}$~~   
 ~~$\vec{B}_s$  is in  $\hat{y}$  direction~~

scattered field  $\vec{E}_s \propto \hat{n} \times (\hat{n} \times \ddot{\vec{p}})$

$\vec{B}_s \propto -\hat{n} \times \dot{\vec{p}}$

The flux  $S = \frac{1}{4\pi} (\vec{E} \times \vec{B})$

~~So the~~ Since  $\vec{p}$  is in  $\hat{x}$  direction there is no wave scattered in  $\hat{x}$  direction, the intensity for  $\hat{y}$ , backward  $\hat{z}$  and forward  $\hat{z}$  direction are all the same.

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How about contribution from the mag. dipole?

good

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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(a)

near  $x=0$

$y''' = 0$

$\therefore y = ax^2 + bx + c$

(b)

$y = Cx^\alpha e^{\beta x}$

~~$x > 1$~~   $x > 1$

The highest order term in  $y'''$

is:  ~~$x^\alpha (\beta x)^3 x^{3(x-1)} e^{\beta x}$~~   $x^\alpha (\beta x)^3 x^{3(x-1)} e^{\beta x}$

the next highest:  $[\alpha (\beta x)^2 + (\alpha + x - 1) (\beta x)^2 + (\alpha + 2x - 2) (\beta x)] x^{\alpha-1} e^{\beta x}$

now clearly:  $\begin{cases} (\beta x)^3 = 1 \\ \alpha + 3(x-1) = 3 + \alpha \\ \alpha + (\alpha + x - 1) + (\alpha + 2x - 2) = 0 \end{cases}$

B

$\therefore \begin{cases} x = 2 \\ \beta = \frac{1}{2} \\ \alpha = -1 \end{cases}$

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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$$\therefore y = \sum_{n=0}^{\infty} a_n x^{6n}$$

~~$a_{n+1} = \frac{a_n}{(6n+4)(6n+5)(6n+6)}$~~

$$a_{n+1} = \frac{a_n}{(6n+4)(6n+5)(6n+6)}, \quad a_0 = 1$$

(d) need only consider large n.

then  $a_{n+1} \approx \frac{a_n}{6^3 n^3}, \quad a_0 = 1$

$$\therefore a_{n+1} = \frac{1}{6^{3(n+1)}} \cdot \frac{1}{(n!)^3}$$

$$\therefore y \approx \sum_{n=0}^{\infty} \frac{1}{6^{3n}} \frac{1}{(n!)^3} x^{6n}$$

no/

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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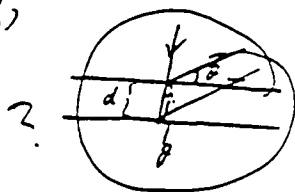
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(4) If you have a ring of superconducting material at its normal state, you apply a magnetic field through the ring, cool it down under the critical temperature, then turn off the field, then the flux through the ring is captured by it. This will not happen for a perfect conductor.  $\times$

a)  $\frac{0.5}{2}$

b)



$$2d \sin \theta = n\lambda$$

another form of this is  $\vec{k} \cdot \vec{k} = \frac{k^2}{2}$ ,  $\vec{k}$  is a reciprocal lattice vector

from  $d \sin \theta = n\lambda$

we see for neighbouring  $n$ ,

$$\sin \theta_2 - \sin \theta_1 = \frac{\lambda}{d}$$

$$\therefore d = \frac{\lambda}{\sin \theta_2 - \sin \theta_1}$$

b)  $1.5/3$

Or we can find all the reciprocal lattice vectors, and remember the smallest reciprocal vector ~~is~~ in one direction has the magnitude  $\frac{2\pi}{d}$ , where  $d$  is the spacing between two nearest lattice planes in the set of planes normalizing in the direction of the reciprocal lattice vector.

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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(a)

The internal energy of the system

$$E = \frac{3}{5} N E_F$$

$$PV = \frac{2}{3} E$$

$$\therefore PV = \frac{2}{5} N E_F$$

$$P = \frac{2}{5} \frac{N E_F}{V}$$

$$\therefore W = \int P dV = \frac{2}{5} N E_F \ln(1.1)$$

Almost but not quite right.  
( $E_F$  changes with  $V$ )

Correct answer is

$$\frac{2}{5} N E_F (1 - 1.1^{-2/3})$$