

Department of Physics  
University of California, San Diego  
La Jolla, California 92093-0319

Written Departmental Examination - Fall, 1991

PART I

Instructions:

Each problem is worth 10 points. This part has 8 problems.

**Problem 1** Describe briefly, with equations or sketches, as needed, the following famous effects or experiments

- a) The Auger effect
- b) Bragg diffraction
- c) Rutherford scattering
- d) The Mössbauer effect
- e) The Stern-Gerlach experiment

**Problem 2**

- a) Consider the function  $f(z) = \sqrt{z}/(1+z^2)$ . Sketch the analytic structure of  $f(z)$  in the complex  $z$ -plane, identifying poles and branch cuts.
- b) Evaluate the integral

$$I = \int_0^{\infty} dx \frac{\sqrt{x}}{1+x^2}$$

using a suitable contour in the complex plane.

**Problem 3**

The density of a sphere of radius  $a$  is

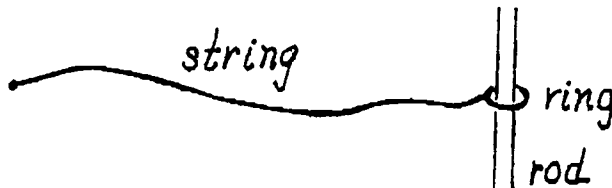
$$\rho(r) = \rho_0 e^{-kr}.$$

There is no mass outside the sphere.

- a) Find the gravitational potential everywhere.
- b) Find the frequency of small oscillations for a particle near the center of the sphere.

### Problem 4

Consider a string of mass  $m$  and length  $L$ , which is stretched under tension  $T$ . One end of the string is fixed, and the other end is tied to a massless ring which slides without friction on a rod. You may ignore gravity.

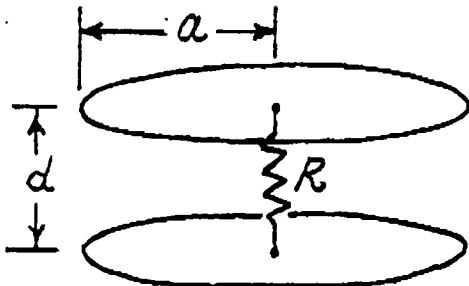


- Find (derive) the equation of motion for the string.
- What are the boundary conditions at the ends of the string?
- Find the normal mode frequencies for small oscillations about the equilibrium position.

### Problem 5

A parallel-plate capacitor has circular disk electrodes of radius  $a$  that are separated by distance  $d$  ( $d \ll a$ ). A resistor of resistance  $R$  connects the plates along their common axis.

At  $t = 0$ , the capacitor is suddenly given charge  $Q$ . Derive an expression for the Poynting vector  $\mathbf{S}$  as a function of position between and outside the plates, for times  $t > 0$ . (Neglect all edge effects).



### Problem 6

A long rectangular bar at temperature  $T$  exerts a tension,  $t(\xi, T)$ , when it is extended an amount  $\xi$  beyond its natural length.

- a) What is the thermodynamic relation between  $dS$ ,  $T$ ,  $dE$ ,  $t(\xi, T)$  and  $d\xi$ , where  $S$  is the entropy and  $E$  is the internal energy of the bar?
- b) What is the relationship between  $(\partial S/\partial \xi)_T$  and  $(\partial t/\partial T)_\xi$  for a quasistatic process?
- c) Let  $t(\xi, T) = b\xi(1 - \gamma/T)$ , where  $b$  and  $\gamma$  are constants. What is the change in internal energy when the bar is stretched from  $\xi = 0$  to  $\xi = \xi_0$  at constant temperature?
- d) What is the sign and magnitude of the heat,  $Q$ , necessary to maintain the bar at constant temperature during this process?

### Problem 7

Assume in a system of volume  $V$  at temperature  $T$  there are excitations which obey Bose statistics with  $\epsilon = bp^{3/2}$ , where  $\epsilon$  is the energy and  $p$  is the momentum. The number of excitations is not conserved.

- a) Derive an expression for the temperature dependence of the heat capacity,  $C$ , of the system. Express your answer in terms of Planck's constant and  $T$ , leaving any integral in dimensionless form.
- b) What would be the temperature dependence of  $C$ , if the system were in a space of arbitrary dimension  $D$ ?

**Problem 8**

A beam of electrons is prepared with all of the electrons in the same spin state. An experimentalist measures the spin in the  $z$  direction, and finds that its average value is 0. Next, the average value of spin in the  $x$  direction is measured, and found to be  $f\hbar$ . Predict the possible value(s) for a measurement of the average spin along the  $y$  direction.

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PART II

**Instructions:**

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**Some Information**

1) Bose condensation temperature:

$$k_B T_c = \frac{h^2}{2\pi m} \left( \frac{n}{2.612} \right)^{2/3}$$

(n = density, m = mass)

2) Quadrupole tensor components for charges  $\{q_\alpha\}$  at positions  $\mathbf{r}_\alpha = (x_1^\alpha, x_2^\alpha, x_3^\alpha)$ :

$$Q_{ij} = \sum_{\alpha} q_{\alpha} (3x_i^{\alpha} x_j^{\alpha} - r_{\alpha}^2 \delta_{ij})$$

## Problem 9

Answer the following:

- a) Describe how you would measure the gravitational constant  $G$ .
- b) Describe how you would measure the dielectric constant of an insulator.
- c) Describe how you would measure the magnetization of a substance.
- d) The 4f electron shell of the trivalent rare earth ion Eu contains six electrons. Using Hund's rules, calculate the spin  $S$ , orbital angular momentum  $L$ , and total angular momentum  $J$  of the ground state.
- e) Describe the origin of the Van der Waals interaction and give a rough derivation of its dependence on distance at large distances.

## Problem 10

A particle is moving under the influence of a periodic impulse. The motion is described by the equation

$$\ddot{x} = \sum_{n=-\infty}^{\infty} A \delta(t-n)$$

with  $A$  a given constant and  $\delta(t)$  the usual Dirac  $\delta$ -function; i.e.,  $\delta = 0$  for  $t \neq 0$

and  $\int_{-\infty}^{\infty} \delta(t) dt = 1$ .

- a) Apply Floquet's Theorem to describe the nature of the motion.
- b) At some initial time  $x$  and  $\dot{x}$  are given and finite. For what range of  $A$  would the motion remain bounded as  $t \rightarrow \infty$ ?

**Problem 11**

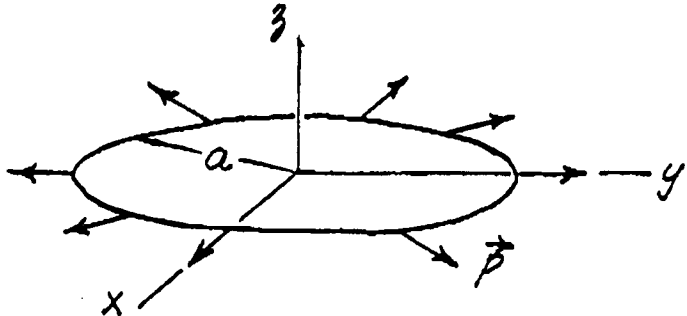
A mass  $m$  is attached to a spring with variable constant  $k(t)$  thus obeying:

$$m\ddot{x} = -k^2(t)x$$

- Write down the Hamiltonian and Lagrangian for the system.
- For  $t < t_0$ ,  $k = k_0$  and the motion is given by  $x = x_0 \sin \omega_0 t$  [ $\omega_0 = k_0/m^{1/2}$ ]. At  $t = t_0$  the spring constant suddenly changes from  $k_0$  to  $k_1$ . What is the change in energy?
- With the same initial conditions as in b) the spring constant for  $t > t_0$  changes "slowly" from  $k_0$  to  $k_1$ . What is the change in energy between the initial and final motion? Give an approximate condition for how slowly the spring constant must change for your result to be valid.

**Problem 12**

- Consider a ring of radius  $a$  on the  $x$ - $y$  plane with a uniform electric dipole density  $\vec{p}$  per unit length pointing in the radial direction:



Calculate the electrostatic potential at position  $\vec{r}$ , and the electric fields along the  $x$  and  $z$  axis, for  $|\vec{r}| \gg a$ . Sketch the electric field lines in the  $x$ - $z$  plane.

- Given a set of  $N$  point particles with no charge and magnetic moment  $\vec{m}$  per particle, describe an arrangement that will give rise to the electric field calculated in (a) and zero magnetic field. Find  $\vec{p}$  in terms of  $N$ ,  $\vec{m}$ ,  $a$ , and any other quantity needed. Assume  $N \gg 1$ .



### Problem 13

In hydrodynamics, the vorticity  $\omega$  is related to the flow  $\mathbf{v}$  by  $\omega = \nabla \times \mathbf{v}$ .  
Taking  $\nabla \cdot \mathbf{v} = 0$

- a) Calculate the flow field induced by a localized vortex  $\omega = \omega_0(\mathbf{x})$ .
- b) Calculate the force of attraction  $F_{1,2}$  between two vortices  $\omega_1, \omega_2$  given that

$$F_{1,2} = \rho_0 \int \mathbf{v}_2 \times \omega_1 d^3x_1$$

with  $\rho_0$  the density.

### Problem 14

Consider a system of spin 1/2 fermions in a box of volume  $V$ . Two fermions of opposite spin can combine to form a boson of spin zero, with binding energy  $\epsilon_b > 0$ . The fermions have mass  $m$  and the bosons have mass  $m_b$  (unrelated to  $m$  and  $\epsilon_b$ ). At infinite temperature there are  $N_\uparrow = N_\downarrow \equiv N_0/2$  fermions of each spin and no bosons. Assume  $N_\uparrow = N_\downarrow$  at all temperatures, and that fermions and bosons behave as ideal gases.

- a) What is the state of the system at  $T = 0$ ?
- b) Find equations that determine the number of fermions and bosons at temperature  $T$ .
- c) Find an equation that determines the critical temperature  $T_c$  of this system, at which a phase transition occurs.
- d) If  $T_{c0}$  is the Bose condensation temperature for the system if all the fermions have combined to form bosons (i.e., in the limit  $\epsilon_b \rightarrow \infty$ ), calculate the deviation  $(T_c - T_{c0})/T_{c0}$  for  $\epsilon_b/T_{c0} \gg 1$ .

- e) Even if you did not solve for (d), answer on physical grounds: Is  $T_C$  larger or smaller than  $T_{CO}$ ? Is  $T_C$  an increasing or decreasing function of  $m/m_b$ ? Justify your answers.

### Problem 15

The proton of a hydrogen atom in the ground state receives an impulse which gives it a velocity  $v$ . The duration  $\tau$  of the impulse is assumed short in comparison both with the electron periods and with  $a/v$  where  $a$  is the Bohr radius. Determine the total probability of excitation and/or ionization of the atom under the influence of such a "jolt".

### Problem 16

The exam committee decided that the interaction Hamiltonian for an atom in an external magnetic field  $\mathbf{B}$  is

$$H_I = -\frac{e}{2Mc} (\mathbf{L} + 3\mathbf{S}) \cdot \mathbf{B}$$

where  $\mathbf{L}$  and  $\mathbf{S}$  are the electron angular momentum and spin, respectively. [NOTE THAT THE HAMILTONIAN INVOLVES  $\mathbf{L} + 3\mathbf{S}$  AND NOT  $\mathbf{L} + 2\mathbf{S}$ .] The  $g$  factor is defined by

$$\langle \ell s j m | H_I | \ell s j m \rangle = -\frac{e\hbar}{2Mc} g m |\mathbf{B}|$$

where  $|\ell s j m\rangle$  is a state with spin  $S^2 = s(s+1)\hbar^2$ , orbital angular momentum  $L^2 = \ell(\ell+1)\hbar^2$ , total angular momentum  $J^2 = j(j+1)\hbar^2$ , and  $J_z = m\hbar$ , with  $\mathbf{J} = \mathbf{L} + \mathbf{S}$ .

- What is  $g$  for a state with  $s = 0$ ?
- What is  $g$  for a state with  $\ell = 0$ ?
- Derive a general formula for  $g$  for a hydrogen atom, and thus determine  $g$  for an atom in a  $2P_{3/2}$  state.

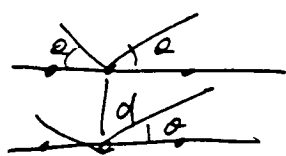
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a. Auger effect: in this phenomenon when an electron of outer shell makes a transition to the inner shell (where there is vacancy), no photon is emitted. Instead another electron get the energy and is excited or emitted.

b. Bragg diffraction: X-ray diffraction of crystal show bright maxima in some direction. They correspond to different crystal plane.



$n\lambda = 2d \sin \theta$   
 ↓  
 maxima.

c) Rutherford scattering: incident charged particles (like  $\alpha$ ) are scattered by nucleus. There is a number of large-angle scatterings (even back-scattering), which wouldn't happen if the atomic charges were distributed uniformly and extendedly (nucleus). So it shows the charge in the atom are concentrated in point-like core.

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$$G(\theta) = \left( \frac{2Ze^2}{E_{inc} 16\pi\epsilon_0} \right)^2$$

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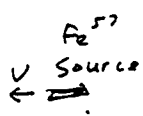
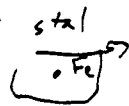
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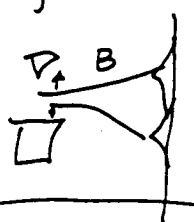
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d). The Mössbauer effect: The  $\gamma$  ray emitted by a nucleus (say  $Fe^{57}$ ) usually can't be absorbed by the same kind of nucleus  $\otimes$ . But if they are put in the crystal, the momentum can be transferred to the whole crystal, which cause negligible energy loss, and the process can happen. Due to the interaction with crystal field, the nuclear energy level may have fine split or shift. This can be compensated by Doppler effect, by moving the source. Thus by measuring the speed at resonance absorption - speed relation we can detect the small shift.



e) Stern-Gerlach experiment: ~~total~~ Silver atom beam (Spin  $1/2$ ) goes through a spatially-varying magnetic field. Due to the interaction with the field the beam will be splitted into two beams, corresponding to different polarization. This shows the existense of half spin particles



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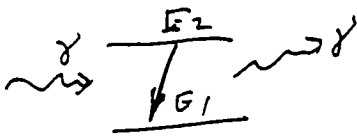
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d) ⊗ The reason is for emission, due to the reciprocal ~~matrix~~ nucleus



$E_\gamma < E_2 - E_1$  ✓ in order for energy and momentum to ~~be conserved~~ be conserved  
 and for absorption in general

$$E_\gamma > E_2 - E_1$$

which shows the emitted  $\gamma$  ray can't excite same kind of nucleus from  $E_1$  to  $E_2$ .

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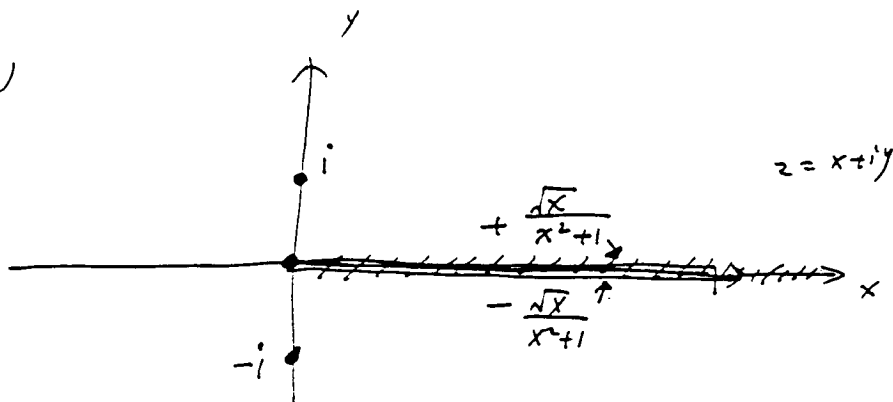
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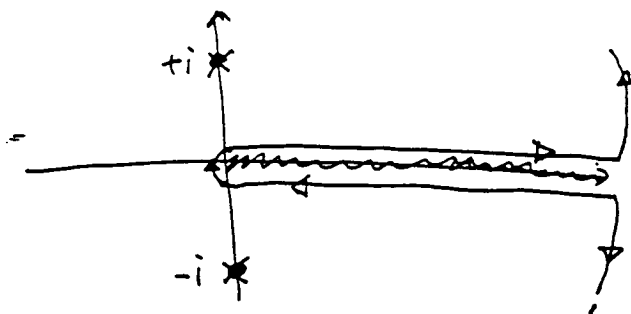
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2) There is a branch cut for  $\sqrt{z}$ , we put it along the real - positive axis. Two poles <sup>are</sup> at  $ti, -i$

$$f(z) = \frac{\sqrt{z}}{(z+i)(z-i)}$$



b).



Let  $f(z) = \frac{\sqrt{z}}{1+z^2}$

above the branch cut

$$f(x) = \frac{\sqrt{x}}{1+x^2}$$

below the branch cut

$$f(x) = -\frac{\sqrt{x}}{1+x^2}$$

On the large ~~circle~~ circle, when radius  $R \rightarrow \infty$ , the integral vanish. Same for the small circle around origin; when radius goes to zero.

Note: If you use additional sheets for this problem, number the pages and staple them together.

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Thus ~~is~~ the integral of the ~~the~~ contour sketched

is

$$I = \oint f(z) dz = \int_0^{\infty} \frac{\sqrt{x}}{1+x^2} dx - \int_0^{\infty} \left( \frac{-\sqrt{x}}{1+x^2} \right) dx$$

$$= 2 \int_0^{\infty} \frac{\sqrt{x}}{1+x^2} dx$$

on the other hand it's equal to the <sup>sum of</sup> residues on the two poles.

$$So \int_0^{\infty} \frac{\sqrt{x}}{1+x^2} dx = \frac{I}{2} = \frac{1}{2} e^{i\pi/2} \left( \frac{e^{i\pi/4}}{(i+i)} + \frac{e^{i\frac{3}{4}\pi}}{(-i-i)} \right)$$

$$= \pi \left( \frac{1}{2} (e^{i\frac{\pi}{2}} - e^{i\frac{3}{4}\pi}) \right)$$

$$= \frac{\pi}{2} (\cos \frac{\pi}{4} - \cos \frac{3}{4}\pi)$$

$$= \frac{\sqrt{2}\pi}{2}$$

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 a). The gravitational ~~force~~<sup>field</sup> is (in analogy with electrical case)


the  
 1) outside sphere :

$$\vec{E}_{\text{grav}} = - \frac{GM}{r^2} \vec{e}_r \quad \checkmark$$

where ~~m~~ m is the total mass

$$\begin{aligned} m &= \int_0^a \rho(r) 4\pi r^2 dr \\ &= 4\pi \rho_0 \int_0^a e^{-kr} r^2 dr \\ &= \frac{4\pi}{k^3} \rho_0 (2 - 2e^{-ka} - 2(ka)e^{-ka} - (ka)^2 e^{-ka}) \end{aligned}$$

Thus if we take potential at infinity to be 0, we have



$$\phi(r) = - \int_{\infty}^r \left( - \frac{GM}{r^2} \right) dr = - \frac{GM}{r} \quad \checkmark$$

2) inside the sphere . the only contribution to the field comes from those inside the r-sphere . other contribution cancels

$$\text{So } \vec{E} = - \frac{Gm(r)}{r^2} \vec{e}_r \quad \checkmark$$

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where  $m(r) = \int_0^r \rho(r') 4\pi r'^2 dr'$

$$= \frac{4\pi}{k^3} \rho_0 (2 - 2e^{-kr} - 2(kr)e^{-kr} - (kr)^2 e^{-kr})$$

$$\int_0^a \varphi(r) = \varphi(a) + \int_r^a \frac{G m(r)}{r^2} dr$$

We can also look at it this way

$$\varphi(r) = - \int \frac{G \rho(r') d^3r'}{|r^2 - r'^2|} = - \int G \rho(r') \int \frac{r'^2}{r'^3} \rho(\cos\theta) d^3r'$$

only two terms survive:  $= - \int_{r < r'} \frac{G \rho_0 e^{-kr'}}{r'^2} d^3r' - \int_{r < r' < a} \frac{G \rho_0 e^{-kr'}}{r'} d^3r'$

$$= - \frac{G \rho_0}{r} \int_0^r e^{-kr'} 4\pi r'^2 dr' - \frac{G \rho_0}{r} \int_r^a e^{-kr'} 4\pi r'^2 dr'$$

$$= - \frac{G \rho_0}{r} \frac{4\pi}{k^3} (2 - 2e^{-kr} - 2(kr)e^{-kr} - (kr)^2 e^{-kr}) - \frac{4\pi G \rho_0}{k^2} (kr(e^{-kr} + 1) - ka(e^{-ka} + 1))$$

b). consider the radial motion only.

when  $r \sim 0$ .

$$m(r) \approx 4\pi \rho_0 \int_0^r e^{-kr'} r'^2 dr'$$

$$\sim 4\pi \rho_0 \int_0^r r'^2 dr' = \frac{4\pi}{3} \rho_0 r^3$$

So the equation of motion for the particle is

$$\ddot{r} = - \frac{G m(r)}{r^2} = - \frac{4\pi}{3} \rho_0 r \quad \omega = \sqrt{\frac{4\pi}{3} G \rho_0}$$

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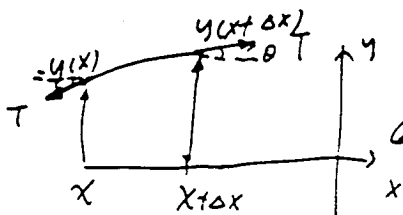
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a) let  $y(x)$  be the displacement at  $x$



Neglecting gravity the force acting on the typical section  $(x, x + \Delta x)$  is

$$\Delta F = T \sin \theta(x + \Delta x) - T \sin \theta(x)$$

when oscillation amplitude is small

$$\sin \theta \sim \theta \sim \tan \theta \sim \frac{dy}{dx}$$

$$\Delta F = T \left( \frac{dy}{dx} \Big|_{x+\Delta x} - \frac{dy}{dx} \Big|_x \right) = T \frac{d^2 y}{dx^2} \Delta x$$

So equation of motion

$$\ddot{y} = \frac{\Delta F}{\Delta m} = \frac{T \frac{d^2 y}{dx^2} \Delta x}{\frac{m}{L} \Delta x} = \frac{TL}{m} \frac{d^2 y}{dx^2}$$

b). at the fixed end ( $x=0$ )  $y(0)=0$ .

at the sliding end ( $x=L$ )  $\frac{dy}{dx} \Big|_{x=L} = 0$

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$$c). \quad \frac{d^2y}{dt^2} = \frac{TL}{m} \frac{d^2y}{dx^2}$$

describing waves on the string.

$$\text{Let } c^2 = \frac{TL}{m}$$

For the normal mode, let  $y(x,t) = A(x) e^{i\omega t}$

$$\frac{TL}{m} \frac{d^2A}{dx^2} + \omega^2 A = 0$$

$$\frac{d^2A}{dx^2} + \frac{\omega^2}{c^2} A = 0$$

These describe the sine (cosine) waves.

$$A = a \sin kx + b \cos kx, \text{ so } \underline{k^2 = \frac{\omega^2}{c^2}}$$

$$x=0. \quad A=0 \Rightarrow b=0$$

$$A = a \sin kx$$

$$\left. \frac{dy}{dx} \right|_{x=L} = 0 \Rightarrow \cos kL = 0 \Rightarrow kL = \left(n + \frac{1}{2}\right)\pi, \quad n = 0, 1, \dots$$

$$\text{So } k = \frac{\left(n + \frac{1}{2}\right)\pi}{L}$$

$$\omega = kc = \frac{\left(n + \frac{1}{2}\right)\pi}{L} \cdot \sqrt{\frac{TL}{m}} = \left(n + \frac{1}{2}\right)\pi \sqrt{\frac{T}{mL}}$$

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Suppose  $Q(t)$  is the charge at time  $t$ , and suppose it's uniformly distributed (neglecting edge effect)  $Q(0) = Q_0$

$Q = CV$  ,  $C$  is capacitance

$$\frac{dQ}{dt} = -I = -\frac{V}{R}$$

So  $C \frac{dV}{dt} + \frac{V}{R} = 0$

or  $V = V_0 e^{-\frac{t}{RC}}$

$V_0 = \frac{Q_0}{C}$

$$V = \frac{Q_0}{C} e^{-\frac{t}{RC}}$$

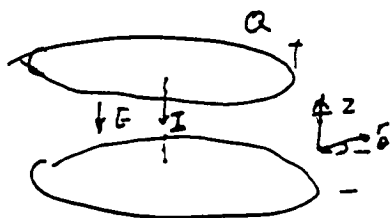
For the plates, the capacitance is

$$C = \frac{Q}{V} = \frac{Q}{E d} = \frac{Q}{\frac{Q}{\epsilon_0} d}$$

$$= \frac{\epsilon_0 S}{d} = \frac{\epsilon_0 \pi a^2}{d}$$

(a) We use cylindrical co-ordinate in this part.

Now in between the plates the  $\vec{E}$  field is



$$E = \frac{V}{d} = \frac{Q_0}{Cd} e^{-\frac{t}{RC}} \quad \text{direct along } \underline{\underline{-\hat{e}_z}}$$

due to the cylindrical symmetry the  $\vec{B}$

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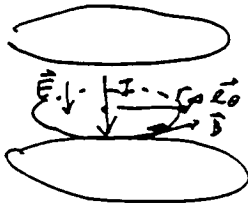
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have the form

Cylindrical coordinate in this part

field should be  $B(r) \hat{e}_\theta$



Since  $\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$

$$\int \nabla \times \vec{B} \cdot d\vec{s} = \int \left( \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{s}$$

$$= \oint \vec{B} \cdot d\vec{l}$$

$c$  - speed of light

Use the circle which centers on the axis of capacitor we

have

$$B(r) \cdot 2\pi r = -\mu_0 I + \frac{1}{c^2} \frac{\partial}{\partial t} (\vec{E} \cdot \hat{e}_z) \pi r^2$$

$$= -\mu_0 I + \frac{1}{c^2} \frac{\partial}{\partial t} \left( \frac{V_0}{d} e^{-\frac{t}{RC}} \right) \pi r^2$$

$$= \frac{V_0}{c^2 d} e^{-\frac{t}{RC}} \pi r^2 - \mu_0 I$$

$$B(r) = \frac{E(t) r}{2c^2 d} - \frac{\mu_0 I}{2\pi r}$$

define  $\tau = RC$  time constant

$$\vec{S} = \vec{E} \times \vec{H} = \frac{V_0}{d} e^{-\frac{t}{RC}} (-\hat{e}_z) \times \frac{B(r)}{\mu_0} (\hat{e}_\theta)$$

$$= \frac{V_0}{d} e^{-\frac{t}{RC}} \frac{B(r)}{\mu_0} \hat{e}_r$$

$$= \left( \frac{E(t) r}{2c^2 \mu_0 d} - \frac{I E(t)}{2\pi r} \right) \hat{e}_r$$

$$E(t) = \frac{V_0}{d} e^{-\frac{t}{RC}}$$

$$I = \frac{V_0}{R} e^{-\frac{t}{RC}}$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score =

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the Problem No. 5

and your Identification No. . 69

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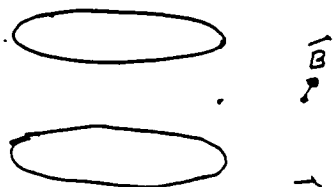
b). outside the plates.

~~at different position~~

at different position we have different kind of behavior of the field. However

In case like left hand side:

We have  $\vec{E} \sim 0$  (neglect edge effect) ~~done so the~~



$$\vec{B} = \frac{1}{2r} \left( \frac{V_0}{c^2 \tau d} e^{-\frac{r}{\tau}} \pi a^2 - \mu_0 I \right)$$

If we go far away from the plate. It looks like an electric dipole with changing moment. We will have dipole radiation.

$$\vec{p} = Qd \vec{e}_z$$

$$\vec{B} = C_1 \frac{\ddot{\vec{p}} \times \vec{r}}{r^2}$$

$$\vec{E} = C_2 \frac{\vec{B} \times \vec{r}}{r}$$

with appropriate constant  $C_1, C_2$

However we know

$$\vec{S} = \frac{2|\dot{\vec{p}}|^2}{3C^3} \vec{e}_r = \frac{2d}{3C^3} |\ddot{Q}|^2 \vec{e}_r = \frac{2d}{3C^3} \left| \frac{Q}{\tau^2} \right|^2 = \frac{2dQ^2}{3C^3\tau^4} \vec{e}_r \quad \times$$

$$\vec{e}_r = \frac{\vec{r}}{r}$$

in Gaussian units.

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 10

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the Problem No. 6 and your Identification No. . 69

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(a) When the bar is extended the work it does is

$$dW = -t d\xi$$

So  $dE = \overset{dQ}{TdS} - dW = TdS + t d\xi$  (for ~~reversible~~ reversible process)

(b) For quasistatic (reversible) process.

~~Since~~  $T = \left(\frac{\partial E}{\partial S}\right)_\xi$   $t = -\left(\frac{\partial E}{\partial \xi}\right)_S$

and  $\frac{\partial^2 E}{\partial S \partial \xi} = \frac{\partial^2 E}{\partial \xi \partial S}$  define  $F = E - TS$  ✓

then  $dF = -SdT + t d\xi$

~~We have~~

so  $\left(\frac{\partial S}{\partial \xi}\right)_T = -\left(\frac{\partial t}{\partial T}\right)_\xi$  ✓

(c)  $S$  can be expressed by  ~~$S = S(T, \xi)$~~   $S = S(T, \xi)$

$dE = TdS + t d\xi$

$= T\left[\left(\frac{\partial S}{\partial T}\right)_\xi dT + \left(\frac{\partial S}{\partial \xi}\right)_T d\xi\right] + t d\xi$

When  $dT = 0$

$dE = \left(T\left(\frac{\partial S}{\partial \xi}\right)_T + t\right) d\xi = \left[-T\left(\frac{\partial t}{\partial T}\right)_\xi + t\right) d\xi$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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$$\Delta E = [T(+\gamma b \xi) + b\{1 - \gamma T\}] d\xi$$

$$= +b\{d\xi\}$$

So  $\Delta E = \int_0^{\xi_0} +b\{d\xi\} = +\frac{b\{\xi_0\}^2}{2}$

d). We only talk about heat transfer. It's <sup>if it is</sup> positive  $\Delta Q$  from outside to bar

$\Delta Q \rightarrow$  from outside to bar  
 $\Delta W \rightarrow$  bar does to the outside

$$\Delta Q = \Delta E + \Delta W$$

$$= \Delta E + \int_0^{\xi_0} T d\xi$$

$$= \int_0^{\xi_0} b\{d\xi\} - \int_0^{\xi_0} T d\xi$$

$$= \gamma b T \int_0^{\xi_0} d\xi = \frac{1}{2} \gamma b T \xi_0^2$$

So the sign is positive in our convention.



PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 10

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and your Identification No. . 69

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a). Since the number of excitations is not conserved (like photon)

$$N(\epsilon) = \frac{\rho(\epsilon)}{e^{\frac{\epsilon}{kT}} - 1}$$

Where  $N(\epsilon)$  is the average number of excitations of energy  $(\epsilon, \epsilon+d\epsilon)$   
 $\rho(\epsilon)$  is density of states

$$\begin{aligned} \rho(\epsilon) \frac{d\epsilon}{\lambda} &= \int_{\epsilon(k)=\epsilon} \frac{V}{(2\pi)^3 \hbar^3} d^3\vec{k} = \int_{\epsilon(k)=\epsilon} \frac{V}{(2\pi)^3 \hbar^3} d^3\vec{p} = \frac{V}{(2\pi)^3 \hbar^3} 4\pi p^2 dp \\ &= \frac{V}{(2\pi)^3 \hbar^3} \frac{4\pi}{3} d(p^3) \\ &= \frac{V}{(2\pi)^3 \hbar^3} \frac{4\pi}{3} d\left(\frac{\epsilon}{b}\right)^2 \\ &= \frac{V}{(2\pi)^3 \hbar^3} \frac{4\pi}{3} \frac{2\epsilon}{b^2} d\epsilon \end{aligned}$$

The total energy is

$$\begin{aligned} U &= \int_0^\infty N(\epsilon) \epsilon d\epsilon \\ &= \int_0^\infty \frac{V}{(2\pi)^3 \hbar^3} \frac{4\pi}{3} \frac{2\epsilon^2}{b^2} \frac{d\epsilon}{e^{\frac{\epsilon}{kT}} - 1} \\ &= \frac{V(kT)^3}{3\pi^2 \hbar^3 b^2} \int_0^\infty \frac{x^2 dx}{e^x - 1} = \frac{2V(kT)^3}{3\pi^2 \hbar^3 b^2} \end{aligned}$$

$$2 = \int_0^\infty \frac{x^2 dx}{e^x - 1}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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$$C = \frac{\partial U}{\partial T} = \frac{2V k^3 T^2}{\pi^2 h^3 b^2} \left( \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \dots \right) \approx \frac{2V k^3 T^2}{\pi^2 h^3 b^2} \frac{1}{e} \quad \checkmark$$

b). If in arbitrary space dimension  $D$

$$\rho(\epsilon) d\epsilon \propto \int_{\epsilon(\vec{h})=\epsilon} d^D \vec{h} \propto d(\epsilon)^D \propto \epsilon^{\frac{2D}{3}}$$

$$U \propto \int \frac{\epsilon \rho(\epsilon)}{e^{\epsilon/kT} - 1} d\epsilon \propto \int \frac{\epsilon d\epsilon^{\frac{2D}{3}}}{e^{\epsilon/kT} - 1} \propto (kT)^{\frac{2D}{3} + 1}$$

$$C \propto \frac{\partial U}{\partial T} \propto kT^{\frac{2D}{3}} \quad \checkmark$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 10

Please insert on page

the Problem No. 8 and your Identification No. 69

suppose the electron is in spin state

$$|s\rangle = a|+\rangle + b|-\rangle \quad |a|^2 + |b|^2 = 1$$

$|+\rangle, |-\rangle$  correspond to  $S_z = \frac{1}{2}, S_z = -\frac{1}{2}$

$$\langle s | S_z | s \rangle = 0 \Rightarrow |a|^2 = |b|^2 \quad |a|^2 = |b|^2 = \frac{1}{2}$$

Now let's write the state to be

~~$S_z = \frac{1}{2}$~~

$$\begin{pmatrix} a \\ b \end{pmatrix}$$

under this representation

$$S_x = \frac{\hbar}{2} \sigma_x \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$S_y = \frac{\hbar}{2} \sigma_y \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\begin{aligned} \langle S_x \rangle &= \langle s | S_x | s \rangle = (a^* \ b^*) \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \\ &= \frac{\hbar}{2} (ab^* + b^*a) = \hbar \frac{1}{2} \checkmark \end{aligned}$$

$$\begin{aligned} \langle S_y \rangle &= \frac{\hbar}{2} (a^* \ b^*) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \\ &= \frac{\hbar}{2} i(b^*a - a^*b) \end{aligned}$$

Now we know  $|a|^2 = |b|^2 = \frac{1}{2}$  suppose  $a = \frac{1}{\sqrt{2}} e^{i\theta}, b = a e^{i\theta}$

then  $\langle S_x \rangle = \frac{\hbar}{2} (e^{-i\theta} |a|^2 + e^{i\theta} |a|^2) = \frac{\cos \theta}{2} \hbar = \frac{\hbar}{2}$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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$$\langle S_y \rangle = \frac{\hbar}{2} i (e^{-i\theta} |a|^2 - e^{i\theta} |a|^2)$$

$$= \frac{\hbar \sin\theta}{2}$$

Since  $\cos\theta = 2f$

$$\sin\theta = \pm \sqrt{1 - (2f)^2} = \pm \sqrt{1 - 4f^2}$$

$$\langle S_y \rangle = \left( \pm \sqrt{1 - 4f^2} \right) \frac{\hbar}{2}$$

✓

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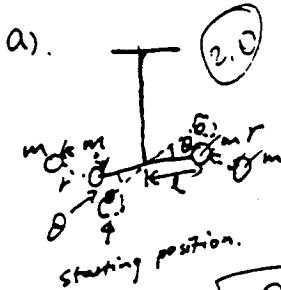
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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

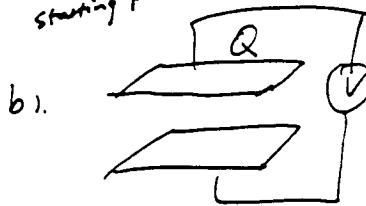
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the Problem No. 9 and your Identification No. 69

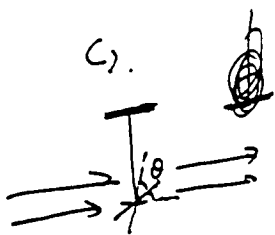


The torque exerted on the ~~object~~ object would turn the string from its original position. The tension in the string caused a torque to balance the one by gravitational force.

$$T = k\theta \quad (T, \text{ torque, } k, \text{ substance constant}) = \frac{2Gm^2}{r^2} \cdot l$$


measure the voltage ~~before~~  $V_1$  in ~~the~~ vacuum and then fill in the space with the dielectric material, measure the voltage again  $V_2$

$$\epsilon = \frac{V_1}{V_2} \quad (2.0)$$



~~put~~ suspend the magnetized substance, put it in an magnetic field perpendicular to its magnetization. (its magnetization ~~direction~~ <sup>axis</sup> can be determined by let it be free to rotate and ~~the final direction it reaches.~~ finally it is going to coincide with  $\vec{B}$  field.) At last it is going to turn around ~~by~~  $\theta$  from by an angle  $\theta$ . The balance equation can be written

$$T = k\theta = |\vec{M} \times \vec{B}| = MB \cos \theta$$

$k$  is a constant depending on the suspending string.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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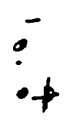
d, 4f.  $l=3$



$S = 3$  maximum  $S$ .

(0.5)

e). molecular are ~~composed of two kind of charges~~ seen as dipole (due to fluctuation of charge)



(1.0)

Van der Waals potential  $\propto \frac{1}{r^6}$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 10

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the Problem No. 10 and your Identification No. 69

a). At time  $t=n$  the particle receives an impulse, and otherwise it's doing uniform motion.  
 Suppose the velocity of the particle ~~is~~ in the period ~~(n, n+1)~~ <sup>time (n, n+1)</sup> ~~(n, n+1)~~ is  $V_n$ .

Then by integrating both side around time  $(n-\epsilon, n+\epsilon)$  we have

$$\dot{x}(n+\epsilon) - \dot{x}(n-\epsilon) = Ax(n)$$

$$\text{Or } V_n - V_{n-1} = Ax(n)$$

Now during the time  $(n-1, n)$  <sup>the</sup> particle moves uniformly:

$$x(n) - x(n-1) = V_{n-1}$$

$$\text{So } V_n = (A+1)V_{n-1} + Ax(n-1)$$

$$x(n) = V_{n-1} + x(n-1)$$

$$\begin{pmatrix} V_n \\ x_n \end{pmatrix} = \begin{pmatrix} A+1 & A \\ 1 & 1 \end{pmatrix} \begin{pmatrix} V_{n-1} \\ x_{n-1} \end{pmatrix}, \quad x_n = x(n)$$

$$= T \begin{pmatrix} V_{n-1} \\ x_{n-1} \end{pmatrix} \quad T \text{ is the transfer matrix.}$$

b). Let's find out the eigen vector and eigenvalue of  $T$ .

They are

$$T \underline{a} = \lambda \underline{a}.$$

$$\lambda_{1,2} = \frac{(A+2) \pm \sqrt{(A+2)^2 - 4}}{2}$$

$$\lambda^2 - (A+2)\lambda + 1 = 0$$

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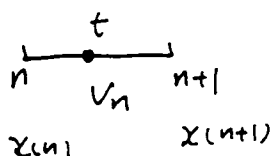
PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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We ~~can~~ can always express the initial condition at time  $t$ , as



So  $x(n) = x(t) - V_n(t-n)$   
 $V_n = V(t)$

$n$  is the largest integer that is equal or smaller than  $t$

So  $x_n$  and  $V_n$  are finite.

Now we can decompose  $\begin{pmatrix} x_n \\ V_n \end{pmatrix}$  by our eigenvectors  $\underline{d}_1, \underline{d}_2$

$$\begin{pmatrix} x_n \\ V_n \end{pmatrix} = a_n \underline{d}_1 + b_n \underline{d}_2$$

and  $\begin{pmatrix} x_{n+m} \\ V_{n+m} \end{pmatrix} = T^m \begin{pmatrix} x_n \\ V_n \end{pmatrix} = T^m (a_n \underline{d}_1 + b_n \underline{d}_2)$   
 $= a_n \lambda_1^m \underline{d}_1 + b_n \lambda_2^m \underline{d}_2$

So  $a_{n+m} = a_n \lambda_1^m$   $b_{n+m} = \lambda_2^m b_n$

In order when  $t \rightarrow \infty$ , or  $m \rightarrow \infty$ , the motion remain bounded

we must have

$$|\lambda_{1,2}|^m \leq 1$$

Since  $\lambda_1 \lambda_2 = 1$  we must have  $|\lambda_1| = |\lambda_2| = 1$

$+ \left| \frac{(A+2) \pm \sqrt{A^2+4A}}{2} \right| = 1$  this can be.

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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the Problem No.  $\frac{010}{\star \quad \star \quad \star}$  and your Identification No.  $\frac{69}{\star \quad \star \quad \star}$

satisfied if

$$A^2 + 4A \leq 0$$

$$\text{Then } \lambda_{\pm} = \frac{(A+2) \pm i\sqrt{|A+4A|}}{2}$$

$$|\lambda|^2 = \frac{(A+2)^2 + |A+4A|}{4} = \frac{(A+2)^2 - (A+4A)}{4} = 1$$

$$\text{So } (A+2)^2 \leq 4$$

$$-2 \leq (A+2) \leq 2$$

$$\underline{-4 \leq A \leq 0}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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the Problem No. 11 and your Identification No. 69

2). Let  $H = \frac{p^2}{2m} + \frac{k^2(t)x^2}{2}$

Then  $\dot{x} = \frac{p}{m}$

$\dot{p} = -k^2(t)x$

So  $m\ddot{x} = -k^2(t)x$

$L = p\dot{x} - H = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}k^2(t)x^2$

b). After to the equation of motion is

$m\ddot{x} = -k_1^2 x$

$\ddot{x} + \omega_1^2 x = 0$        $\omega_1^2 = \frac{k_1^2}{m}$

and  $x_0 = A \sin \omega_1 t + B \cos \omega_1 t$

Since at  $t = t_0$

$$\begin{cases} x = A \sin \omega_1 t_0 + B \cos \omega_1 t_0 = x_0 \sin \omega_1 t_0 \\ \dot{x} = \omega_1 A \cos \omega_1 t_0 - \omega_1 B \sin \omega_1 t_0 = \omega_0 x_0 \cos \omega_1 t_0 \end{cases}$$

Solve the equations, we have

$A = \frac{\omega_1 x_0 \sin \omega_1 t_0 \sin \omega_1 t_0 + \omega_0 x_0 \cos \omega_1 t_0 \cos \omega_1 t_0}{\omega_1 \cos^2 \omega_1 t_0 + \omega_0 \sin^2 \omega_1 t_0}$

$B = \frac{\omega_1 x_0 \sin \omega_1 t_0 \cos \omega_1 t_0 - \omega_0 x_0 \cos \omega_1 t_0 \sin \omega_1 t_0}{\omega_1 \cos^2 \omega_1 t_0 + \omega_0 \sin^2 \omega_1 t_0}$

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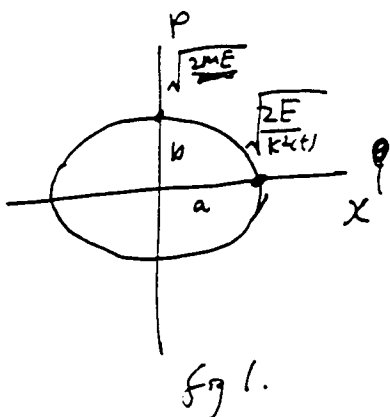
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C). In this case the process is adiabatic and the adiabatic invariant is

$$I = \oint p dx$$

$$= \oint \sqrt{2m(E - \frac{1}{2}k^2x^2)} dx$$



The orbit in phase space is like fig 1. I is the area enclosed.

$$I = S = \pi ab$$

$$= \pi \sqrt{\frac{2E}{k(t)} \cdot 2mE}$$

$$= 2\pi \sqrt{m} \frac{E}{k(t)}$$

So  $E \propto k(t)$

So at last  $E_1 = \frac{k_1}{k_0} E_0$

$\Delta E = (\frac{k_1}{k_0} - 1) E_0$

C) The change in one cycle of oscillation should be small

$\Delta T (k^2(t))' \ll k^2(t)$

$\frac{k'(t)}{k(t)} \ll \frac{1}{2T}$

$T = \frac{2\pi}{\omega} = \frac{2\pi m^{1/2}}{k}$

So  $k'(t) \ll \pi m^{1/2}$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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2) 

The potential of a dipole moment  $\vec{P}$  is

$$\phi = \frac{\vec{P} \cdot \vec{r}}{r^3} \quad (\text{Gaussian unit})$$

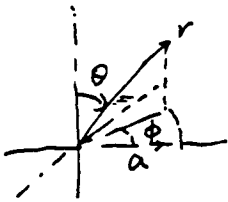
So at position  $\vec{r}$

$$\phi = \int \frac{\vec{P}(\vec{r}') \cdot (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} d\ell'$$

The integration is along the ring.  $\vec{P} = P \frac{\vec{r}'}{r'}$

$$\phi = \int \frac{P}{r'} \frac{\vec{r}' \cdot (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} d\ell'$$

When  $|\vec{r}| \gg a = |\vec{r}'|$   $\frac{1}{|\vec{r} - \vec{r}'|^3} = \frac{1}{r^3} - \vec{r}' \cdot \nabla \frac{1}{r^3} = \frac{1}{r^3} + \frac{3\vec{r}' \cdot \vec{r}}{r^5}$



$$\phi = \int \frac{P}{r'} \vec{r}' \cdot (\vec{r} - \vec{r}') \left( \frac{1}{r^3} + \frac{3\vec{r}' \cdot \vec{r}}{r^5} \right) d\ell' \quad |\vec{r}'| = r' = a$$

$$= \int \left[ \frac{P}{r'} \left( \frac{\vec{r}' \cdot \vec{r}}{r^3} + \frac{3(\vec{r}' \cdot \vec{r})^2}{r^5} \right) - Pr' \left( \frac{1}{r^3} + \frac{3\vec{r}' \cdot \vec{r}}{r^5} \right) \right] d\ell'$$

$$\vec{r}' \cdot \vec{r} = r' r (\sin\theta_r \cos\phi_r \cos\phi_{r'} + \sin\theta_r \sin\phi_r \sin\phi_{r'})$$

$$d\ell = a d\phi_{r'}$$

So after integration only two terms remains

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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the Problem No. 12

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$$\begin{aligned} \phi &= \frac{3P}{r^5} \int_0^{2\pi} (\vec{r} \cdot \vec{r}')^2 d\phi_{r'} - \int_0^{2\pi} \frac{P a}{r^3} a d\phi_{r'} \\ &= \frac{3Pa^2}{r^5} \int_0^{2\pi} (x \cos \phi_{r'} + y \sin \phi_{r'})^2 d\phi_{r'} - \frac{2\pi P a^2}{r^3} \\ &= \frac{3Pa^2}{r^5} \pi (x^2 + y^2) - \frac{2\pi P a^2}{r^3} \\ &= \frac{3\pi P a^2}{r^5} r^2 - \frac{2\pi P a^2}{r^3} \end{aligned}$$

~~Along z axis~~

$$\begin{aligned} \vec{E} = -\nabla \phi &= -\frac{3\pi P a^2}{2} \left( \nabla \frac{1}{r^3} - \nabla \frac{z^2}{r^5} \right) \\ &= \frac{3\pi P a^2}{2} \frac{z}{r^5} \end{aligned}$$

$$\phi = \left( \frac{3\pi P a^2}{2} - 2\pi P a^2 \right) \frac{1}{r^3} - \frac{3\pi P a^2}{2} \frac{z^2}{r^5}$$

$$\begin{aligned} E = -\nabla \phi &= \left( \frac{2\pi P a^2}{2} \right) - \pi P a^2 \nabla \frac{1}{r^3} - 3\pi P a^2 \nabla \frac{z^2}{r^5} \\ &= \pi P a^2 \frac{3\vec{r}}{r^5} - 3\pi P a^2 \frac{2z\vec{e}_z}{r^5} + 3\pi P a^2 z^2 \frac{5\vec{r}}{r^7} \end{aligned}$$

So along x axis:  $\vec{E} = \frac{3\pi P a^2}{r^4} \vec{e}_x$  along z axis:  $\vec{E} = \frac{12\pi P a^2}{r^4} \vec{e}_z$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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the Problem No. 12 and your Identification No. 69

b).



They are on a ring that is rotating at constant angular velocity  $\omega$ .

Since the  $\vec{B}$  field produced (in the <sup>instantaneous</sup> rest frame) by  $\vec{m}$  one particle

$$\vec{B} = \frac{3(\vec{n} \cdot \vec{m})\vec{n} - \vec{m}}{r^3}$$

In lab frame  $\vec{E} = -\vec{v} \times \vec{B} = \frac{3(\vec{n} \cdot \vec{p})\vec{n} - \vec{p}}{r^3}$  ( $\vec{v}$  is the speed of  $\vec{m}$  ( $\vec{v}$ ))


We want it to look like as if generated by dipole  $\vec{p}$

So we have  $\vec{p} = -\vec{v} \times \vec{m}'$

$m' = \frac{N\vec{m}}{2\pi a}$  is the density

$\vec{v} = \vec{\omega} \times \vec{r} = -rav \vec{e}_\theta$

There is no B field since each moment can be seen

as an  small circuit of circulating currents. At such arrangement they cancel each other exactly



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2).  $\nabla \cdot \vec{V} = 0 \Rightarrow \vec{V} = \nabla \times \vec{A}$  ( $\vec{A}$  is some kind of vector potential)

5.  $\vec{\omega} = \nabla \times (\nabla \times \vec{A})$

$= \vec{e}_i \epsilon_{ijk} \partial_j \epsilon_{kij'} \partial_{i'} A_{j'}$

$= \vec{e}_i \delta_{ii'} \delta_{jj'} \partial_j \partial_{i'} A_{j'} - \vec{e}_i \delta_{ij'} \delta_{j'i'} \partial_j \partial_{i'} A_{j'}$

$= \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

Now  $\vec{A}' = \vec{A} + \nabla \chi$  won't change  $\vec{\omega}$ , so we can always make a choice to make  $\nabla \cdot \vec{A}' = 0$

( by choosing  $\nabla^2 \chi + \nabla \cdot \vec{A} = 0$   
 $\chi(\vec{r}) = \frac{1}{4\pi} \int \frac{\nabla \cdot \vec{A}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r'$  )

So we assume  $\nabla \cdot \vec{A} = 0$

So  $\nabla^2 \vec{A} = -\vec{\omega}$

$\vec{A} = \frac{1}{4\pi} \int \frac{\vec{\omega}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r'$

$\vec{V} = \nabla \times \vec{A} = \frac{1}{4\pi} \int \frac{\vec{\omega}(\vec{r}') \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} d^3r'$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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b).  $\vec{v}_2(\vec{x}_1) = \frac{1}{4\pi} \int \frac{\vec{\omega}_2(\vec{x}_2) \times (\vec{x}_1 - \vec{x}_2)}{|\vec{x}_1 - \vec{x}_2|^3} d^3\vec{x}_2$

So  $F_{12} = \frac{\rho_0}{4\pi} \iint \frac{(\vec{\omega}_2(\vec{x}_2) \times (\vec{x}_1 - \vec{x}_2)) \times \vec{\omega}_1(\vec{x}_1)}{|\vec{x}_1 - \vec{x}_2|^3} d^3\vec{x}_1 d^3\vec{x}_2$   
 $= \frac{\rho_0}{4\pi} \iint \frac{(\vec{\omega}_1(\vec{x}_1) \cdot \vec{\omega}_2(\vec{x}_2))(\vec{x}_1 - \vec{x}_2) - \vec{\omega}_1(\vec{x}_1) (\vec{\omega}_2(\vec{x}_2) \cdot (\vec{x}_1 - \vec{x}_2))}{|\vec{x}_1 - \vec{x}_2|^3} d^3\vec{x}_1 d^3\vec{x}_2$

Now the second term vanishes because

$\int \int \frac{\vec{\omega}_2(\vec{x}_2) (\vec{\omega}_1(\vec{x}_1) \cdot (\vec{x}_1 - \vec{x}_2))}{|\vec{x}_1 - \vec{x}_2|^3} d^3\vec{x}_1 d^3\vec{x}_2$   
 $= - \int \int \vec{\omega}_2(\vec{x}_2) (\vec{\omega}_1(\vec{x}_1) \cdot \nabla_{\vec{x}_1} \frac{1}{|\vec{x}_1 - \vec{x}_2|}) d^3\vec{x}_1 d^3\vec{x}_2$   
 $= - \int \int \vec{\omega}_2(\vec{x}_2) \nabla_{\vec{x}_1} \cdot \frac{\vec{\omega}_1(\vec{x}_1)}{|\vec{x}_1 - \vec{x}_2|} d^3\vec{x}_1 d^3\vec{x}_2 \quad (\text{since } \nabla \cdot \vec{\omega} = 0)$   
 $= \int \vec{\omega}_2(\vec{x}_2) \left( \int_S \frac{\vec{\omega}_1(\vec{r}_1)}{|\vec{x}_1 - \vec{x}_2|} dS \right) d^3\vec{x}_2$

Since the sources are localized ~~the~~ the surface integral vanishes

So:  $F_{12} = \frac{\rho_0}{4\pi} \iint \frac{(\vec{\omega}_1 \cdot \vec{\omega}_2)(\vec{x}_1 - \vec{x}_2)}{|\vec{x}_1 - \vec{x}_2|^3} d^3\vec{x}_1 d^3\vec{x}_2$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 4

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the Problem No. 14 and your Identification No. 69

a). At  $T=0$ , there is no fermion. All fermions combine to bosons.  
They are all at state  $\vec{k}=0$

b). Suppose the chemical energy for fermions and bosons are

When they are balanced  $\mu_f = \mu_b$   
 $2\mu_f = \mu_b = \epsilon_b$

$$N_f = \int \frac{2g(\epsilon) d\epsilon}{e^{(\epsilon - \mu_f)/kT} + 1}$$

$$N_b = \int \frac{g(\epsilon) d\epsilon}{e^{(\epsilon - \mu_b)/kT} + 1} + N_{b0}$$

where  $g(\epsilon) d\epsilon = \int_{\epsilon = \frac{p^2}{2m}} \frac{V d^3p}{(2\pi)^3 \hbar^3} = \frac{V 4\pi p^2 dp}{(2\pi)^3 \hbar^3}$

$2 \frac{2m}{\hbar^2}$

$$= \frac{V 4\pi (2m)^{\frac{3}{2}} \frac{1}{2} \epsilon^{\frac{1}{2}} d\epsilon}{(2\pi)^3 \hbar^3}$$

$$= \frac{(2m)^{\frac{3}{2}} V \epsilon^{\frac{1}{2}} d\epsilon}{4\pi^2 \hbar^3}$$

$$N_f + N_b = N_0$$

So the equations determining the numbers are

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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the Problem No.  $\frac{14}{\star}$  and your Identification No.  $\frac{69}{\star}$

$$\left\{ \begin{aligned} \mu_f &= \mu_b + \epsilon_b \\ N_f &= \int \frac{g(\epsilon) d\epsilon}{e^{(\epsilon - \mu_f)/kT} + 1} \\ N_b &= \int \frac{g(\epsilon) d\epsilon}{e^{(\epsilon - \mu_b)/kT} + 1} + N_{b0} \\ N_f + N_b &= N_0 \end{aligned} \right.$$

where  $N_{b0}$  is the condensation number at  $k=0$

Get.  
C). (

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 6

Please insert on each page

the Problem No. 15 and your Identification No. 69 ☆ ☆ ☆ ☆ ☆ ☆ ☆ ☆

Since  $\tau \ll T$   $\tau \ll \frac{a}{v}$

Immediately after the impulse the electron <sup>is</sup> still ~~at~~ at the same distribution as before. Since  $m_e \ll M_p$  no matter what

happens we can assume that the proton will continually move at the speed  $\vec{v}$  and so do the center of mass.

So if we go to the frame which is moving at a relative speed  $v$  ~~regard~~ to the original one the ~~state~~ <sup>wave function</sup> of the electron at  $t=0$  (just after the impulse) will be found to be

$$|\phi_0'\rangle$$

and the probability it remained ~~to be~~ <sup>in the ground state is</sup>

$$P = |\langle \phi_0 | \phi_0' \rangle|^2$$

where  $\phi_0$  is the ground state of the hydrogen in its rest frame.

So we would like to know what  $|\phi_0'\rangle$  is.

Now in the ~~the~~-original frame  $|\phi_0'\rangle$  is just  $\phi_0$ . After change of co-ordination we know that  $|\vec{p}\rangle \rightarrow |\vec{p}-m\vec{v}\rangle$

$$|\phi_0'\rangle = \int |\vec{p}'\rangle \langle \vec{p} | \phi_0 \rangle \frac{d^3p}{(2\pi)^3}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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the Problem No. 15 and your Identification No. 69  
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$$\langle \phi_0 | \phi_0' \rangle = \int \langle \phi_0 | \vec{P}' \rangle_{\text{new}} \langle \vec{P} | \phi_0 \rangle_{\text{old}} \frac{d^3 \vec{P}}{(2\pi)^3}$$

~~new and old~~ the subscripts show in which co-ordination to calculate

$$\langle \vec{P} | \phi_0 \rangle_{\text{old}} = \int e^{-i\vec{P}\vec{x}} \phi_0(\vec{x}) d^3\vec{x}$$

$$\langle \phi_0 | \vec{P}' \rangle_{\text{new}} = \int e^{+i\vec{P}'\vec{x}'} \phi_0^*(\vec{x}') d^3\vec{x}'$$

$$\langle \phi_0 | \phi_0' \rangle = \int \phi_0^*(\vec{x}') e^{-i\vec{P}\vec{x}} e^{i(\vec{P}-m\vec{v})\vec{x}'} \phi_0(\vec{x}) d^3\vec{x} d^3\vec{x}' \frac{d^3P}{(2\pi)^3}$$

$$= \int \phi_0^*(\vec{x}') \phi_0(\vec{x}) e^{-i m \vec{v} \vec{x}'} d^3\vec{x}'$$

?

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 10

Please insert on each page

the Problem No. 16 and your Identification No. 69

a) ~~Choose  $\vec{B}$  to be along z axis~~

for  $s=0$ ,  $\vec{J} = \vec{L}$

$g = 1$  ✓

b) for  $l=0$ ,  $\vec{J} = \vec{S}$

$g = 3$  ✓

c)  $\langle l s j m | H_z | l s j m \rangle = \frac{e\hbar}{2m c} \langle l s j m | (\vec{L} + 3\vec{S}) \cdot \vec{B} | l s j m \rangle$

In analogy with the normal case, we assume that:  $\langle \vec{S} \cdot \vec{B} \rangle = \frac{\langle \vec{S} \cdot \vec{J} \times \vec{J} \cdot \vec{B} \rangle}{J^2} = -\frac{e\hbar}{2m c} m |B| - \frac{e}{2m c} \langle l s j m | 2\vec{S} \cdot \vec{B} | l s j m \rangle$

$$g = 1 + \frac{2\langle \vec{S} \cdot \vec{J} \rangle}{J^2} = 1 + \frac{s^2 + J^2 - L^2}{J^2}$$

$$= 1 + \frac{s(s+1) + j(j+1) - l(l+1)}{j(j+1)}$$

for  $2P_{3/2}$   $s = \frac{1}{2}$ ,  $j = \frac{3}{2}$ ,  $l = 1$

$g = 1 + \frac{\frac{3}{4} + \frac{15}{4} - 2}{\frac{15}{4}} = \frac{5}{3}$  ✓

Note: If you use additional sheets for this problem, number the pages and staple them together.

good