

DEPARTMENT OF PHYSICS
University of California, San Diego
La Jolla, California 92093-0319

Written Departmental Examination - Spring, 1989-90

PART I

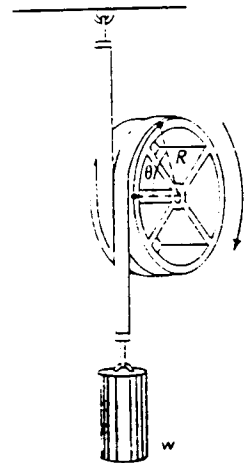
Instructions

Each problem is worth 10 points. You are to work problems 1-8 plus either problem 9 or problem 10.

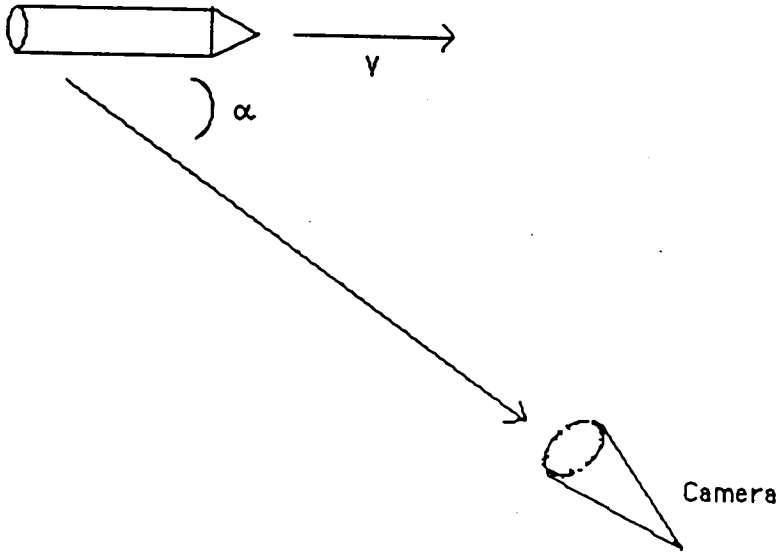
Problem 1

A power brake invented by Lord Kelvin consists of a strong, flexible belt wrapped once around a spinning flywheel. One end of the belt is fixed to an overhead support; the other end carries a weight, W . The coefficient of kinetic friction between the belt and the wheel is μ_k . The radius of the wheel is R , and its angular velocity is ω .

- Find the tension in the belt as a function of the angle of contact, θ .
- Find the net friction torque on the flywheel.
- Find the power dissipated by friction.

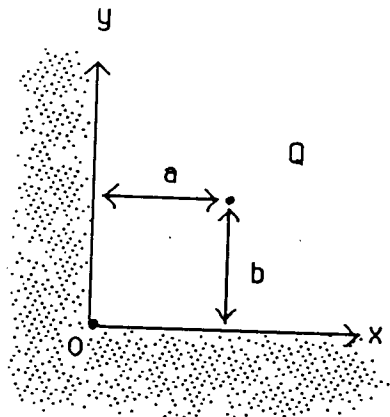


An object moving with velocity $v \lesssim c$ is photographed by a stationary, distant camera. The object has rest length L_0 , and the angle between the direction of motion and the direction to the camera is α (see figure).



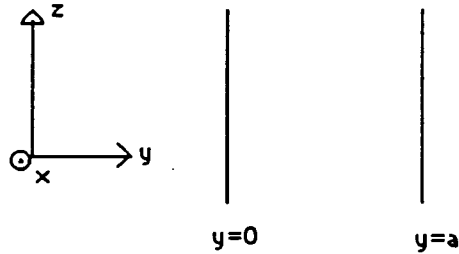
Find the length of the object in the photograph.

Problem 3



A point charge of magnitude Q is placed at a distance, a , from a conducting plate, and b , from a second, perpendicular plate (see figure).

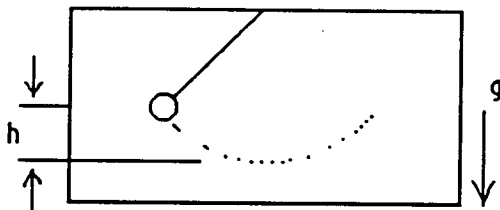
A waveguide consists of two infinite, perfectly conducting plates separated in the y -direction by a distance a (see diagram).



- a) Find the form of the electric field for a TE mode ($\vec{E} // \hat{x}$) propagating in the z -direction, with frequency ω .
- b) For $\omega = 5\pi c/2a$, find the k_z values that propagate.
- c) What is the surface current on the conductor in terms of ω , \vec{k} and E_0 ?

Problem 5

- i) Consider a two-level system with energy states ϵ and $\epsilon + \Delta$ ($\Delta \geq 0$). Compute the partition function and the free energy.
- ii) Derive an expression for the specific heat, $C(T)$. Sketch your result.
- iii) Some glassy systems may be modeled as a set of two level systems with a distribution, $P(\Delta)$, of "barrier heights" Δ . [The distribution of energies, ϵ , is irrelevant.] The free energy is then the sum of contributions from all the individual two-level systems. If $P(\Delta)$ is broad and flat on the scale of kT , what is the temperature dependence of the specific heat of such a "glass" at low temperatures?



thermally isolated at a temperature, T , in a gravitational field of strength, g , and has a total heat capacity, C . The ball is initially at a height, h , above its rest position, as shown. Then the ball is released, and a new equilibrium is reached. (Assume that $Mgh \gg kT$.)

a) What is the increase, ΔT , in the temperature of the system?

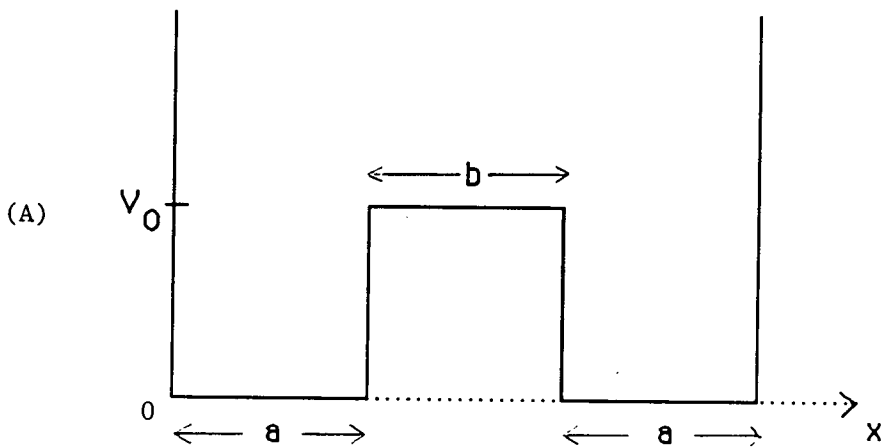
b) What is the change in entropy of the system?

c) What is the probability, P , that the ball will spontaneously return to its original position?

d) Evaluate P for $\Delta T/T \ll 1$, $Mgh = 10^5$ ergs, and $T = 300^\circ\text{K}$.

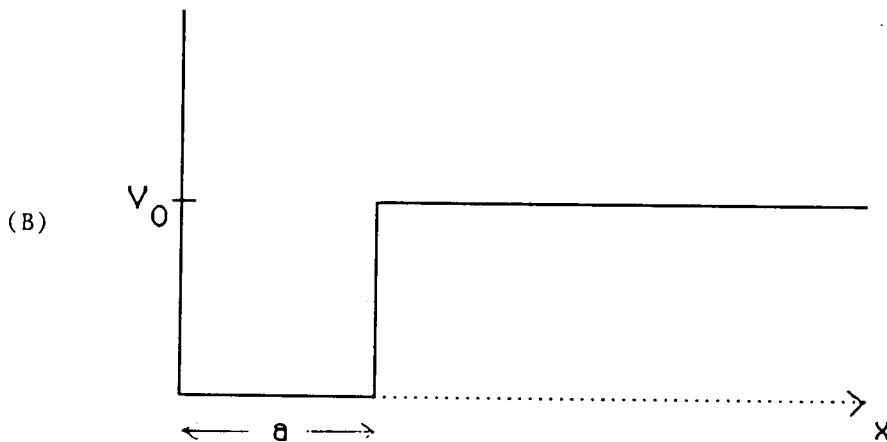
Problem 7

A charged particle of mass m and charge q moves in a harmonic oscillator potential $V(x) = 1/2 kx^2$ in one dimension. In addition, a constant electric field $\vec{E} = E_0 \cdot \hat{x}$ acts on the particle where \hat{x} is the unit vector along the x-direction. What are the quantum mechanical energy levels and stationary wavefunctions of the particle?



a) Draw qualitatively the wavefunctions of the first two energy eigenstates when the ground state energy is much smaller than V_0 .

b) Show that in the limit $b \rightarrow \infty$ one of the two lowest energy eigenvalues will approach the ground state energy E_0 of the potential in Fig. (B) from below, and the other eigenvalue will approach E_0 from above.



Calculate the energy levels and wavefunctions approximately in the limit

$$E \ll V_0, \quad \frac{2mV_0}{\hbar^2} b^2 \ll 1$$

- Define the following properties of a solid, and explain how each can be measured:
(a) electrical resistivity; (b) magnetic susceptibility; (c) specific heat; (d) thermal conductivity.

Problem 10

Evaluate the following integrals:

- (a) $\int_0^{\infty} e^{-\alpha x} x^{\beta} dx$. For what range of β is your answer valid?
- (b) $\oint_c dz \frac{e^{ikz}}{z^3}$. Here the contour is a (counterclockwise) unit circle around the origin.
How would the answer depend on the radius of the circle?
- (c) $\int_0^{\infty} e^{-\beta x^2} dx$. How does the answer to this part relate to (a)?
- (d) $\int_0^{\infty} \exp\{-(x^2 + \alpha/x^2)\} x dx$. Give an approximate answer valid for $\alpha \gg 1$.
- (e) $\oint_c \exp\{-(z^2 + \alpha/z^2)\} z dz$, where the contour is the unit circle given in part (b). If you can't evaluate this integral exactly, give an approximate answer for $\alpha \ll 1$.

PART II

Instructions

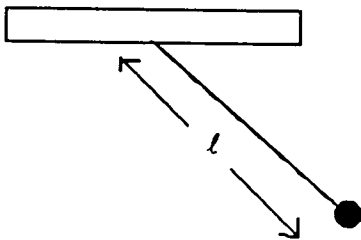
Each problem is worth 10 points. You are to work problems 11-18 plus either problem 19 or problem 20.

Problem 11

- a) Write down the Hamiltonian for a non-relativistic electron that moves in the magnetic field $\mathbf{B} = \frac{\hat{z}B_0x}{L}$, where B_0 and L are constants.
- b) Obtain three constants of the motion for the electron.
- c) Suppose that the electron initially is ejected from the origin with velocity $\mathbf{v} = v_0\hat{x}$. Sketch the projection on the (x,y) -plane of the subsequent orbit. Also determine an integral expression for $y = y(x)$. What can you say about $z = z(x)$?

Problem 12

Consider a simple model of a swing, consisting of a pendulum whose length ℓ can be



Consider a gas of microscopic conducting spheres of radius a and density n_0 . An electromagnetic wave, with $\vec{k} = k\hat{y}$ and $ka \ll 1$, is incident on a half-space of such spheres ($y > 0$). Assume the electric field of the wave is oriented in the \hat{z} direction.

- a) Consider the scattering from a single sphere, and calculate the radiated power into solid angle $d\Omega$.
- b) Using your result from a), find the extinction length for propagation of the electromagnetic wave through the medium.

Problem 14

Consider a non-relativistic charged particle with charge q following a linear trajectory and interacting with a plasma with (complex) dielectric constant $\epsilon(\omega)$.

- a) Find the (Fourier transformed) electrostatic potential $\phi(k, \omega)$.
- b) Write down an integral expression for the rate at which the particle loses energy to the plasma.

Problem 15

A system of N ($N \gg 1$) non-interacting classical particles of mass m are in a container of volume V . Assuming that the particles are distributed in phase space via the microcanonical ensemble with energy E :

- a) Up to a normalization constant, find the probability that particle 1 has momentum within a small box of size d^3p_1 around p_1 .
- b) Find the limiting form of this distribution as $N \rightarrow \infty$, and thereby identify the temperature as a function of E and N .

Maxwell-Boltzmann statistics) and an adsorbed gas on a substrate.

- a) Calculate the chemical potential of the free gas, assuming temperature T , mass m and mean pressure \bar{p} .
- b) Calculate the chemical potential of the adsorbed atoms, assuming that the energy of the adsorbed atom is $\epsilon = \vec{p}^2/2m - \epsilon_0$, where \vec{p} is the momentum in the plane.
- c) By equating the two chemical potentials, find the mean number of adsorbed atoms per unit area as a function of \bar{p} and T .

Problem 17

Consider the scattering of a spin- $\frac{1}{2}$ particle by the spin-orbit potential

$$V = \frac{e^{-\mu r}}{r} \vec{L} \cdot \vec{S} + \text{hermitian conjugate.}$$

If the initial particle has its spin polarized along the momentum direction, find the differential cross-section for spin flip scattering, where the spin of the outgoing particle is anti-parallel to the final momentum. Use the Born approximation.

Problem 18

Consider a quantum system with two energy levels separated by the energy difference, ΔE . At time $t = 0$, we turn on a time-dependent perturbation which connects these two states via matrix elements

$$\langle 1 | V | 2 \rangle = \langle 2 | V | 1 \rangle^* = \gamma e^{i\omega t} \quad (\gamma \text{ real}).$$

Using lowest order perturbation theory, find the probability that, given the system in the ground state at $t = 0$, the system will be in the excited state at time t .

By solving the time-dependent Schrödinger equation, find the exact answer for this

Problem 19

The diffusion equation

$$\frac{\partial U}{\partial t} = \alpha \nabla^2 U + \beta U \quad (1)$$

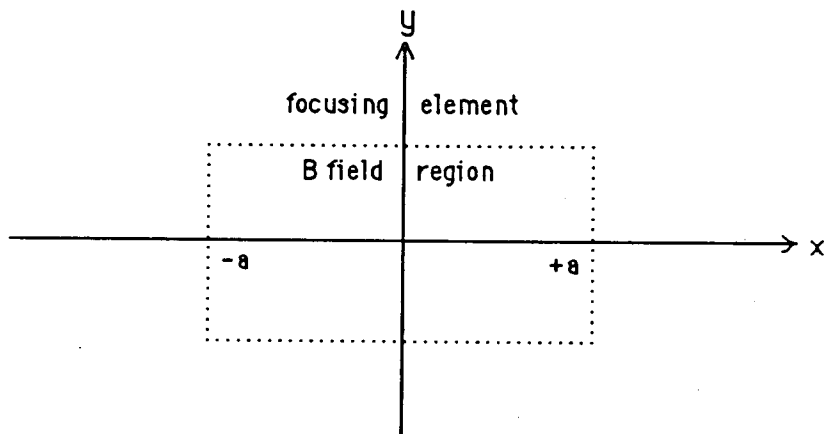
provides a crude model to describe the flow of neutron density $U(t, x, y, z)$ in a block of uranium. Here, α is the diffusion constant, and β governs the rate at which free neutrons are created through collision.

- a) Use separation of variables to obtain a solution in a cube of size $D : 0 < x, y, z < D$, given an arbitrary distribution $U(0, x, y, z) = \Phi(x, y, z)$ and the boundary condition that $U \equiv 0$ on the surface of the cube for all $t \geq 0$.

- b) Use the solution of part a) to determine the critical size, D_c , of such a cube such that U will grow exponentially with time if $D > D_c$.

magnetic field, $B(x,y,z)$. The trajectory of the charged particle can be simulated and measured by appropriately suspending a flexible, massless, wire carrying a current, i , under tension, T . Show that there is a relationship between i, T, q and p , for which the trajectory of the charged particle is the same as the shape of the wire, and derive this expression.

- (b) Using such a suspended wire, describe how you would measure the focal length of a solenoid, which acts as a magnetic lens, on a beam of particles of momentum p , (initially parallel to the x direction), and charge, q . Describe how and where you would constrain the wire and provide the tension, and what you would measure to determine the focal length. (Notes: For this part, you must think of how you would physically set up and carry out the measurement. You do not have to have answered (a) correctly. It will help to refer to the figure.)



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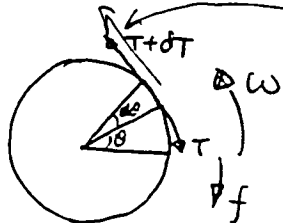
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Solution: a) $dN(\theta) = \frac{(T+dT) - T}{2} \frac{d\theta}{2} + T \frac{d\theta}{2} \approx T d\theta$



$$T+dT - T = \mu_k dN(\theta)$$

$$+ \frac{dT}{d\theta} = \mu_k d\theta \times dT = -d[\text{friction}]$$

$$\Rightarrow T = T_0 e^{+\mu_k \theta}$$

3

∴ at $\theta=0$, $T = W \Rightarrow T = W e^{+\mu_k \theta}$

3 b) $\tau = \int \mu_k T R d\theta = \mu_k R W \int_0^{2\pi} e^{+\mu_k \theta} d\theta = WR (1 + e^{+2\pi})$

2 c) $P = \omega \cdot \tau = WR\omega (1 + e^{+2\pi \mu_k}) = WR\omega (e^{2\pi \mu_k} - 1)$

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~~$L = \frac{L_0}{(1 - \frac{v}{c} \cos \alpha)} \gamma = \frac{L_0}{1 - \frac{v}{c} \cos \alpha} \sqrt{1 - \frac{v^2}{c^2}}$~~

~~if the length in the photo~~

~~is $L = \frac{L_0}{1 - \frac{v}{c} \cos \alpha} \sqrt{1 - \frac{v^2}{c^2}}$~~

$L_0 = \gamma (L - \frac{Lv \cos \alpha}{c}) \Rightarrow L = \frac{L_0}{\gamma (1 - \frac{v \cos \alpha}{c})} = \frac{L_0 \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v \cos \alpha}{c}}$

\Rightarrow the length in the photo

$L = \frac{L_0 \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c} \cos \alpha}$

except for the $\sin \alpha$ projection

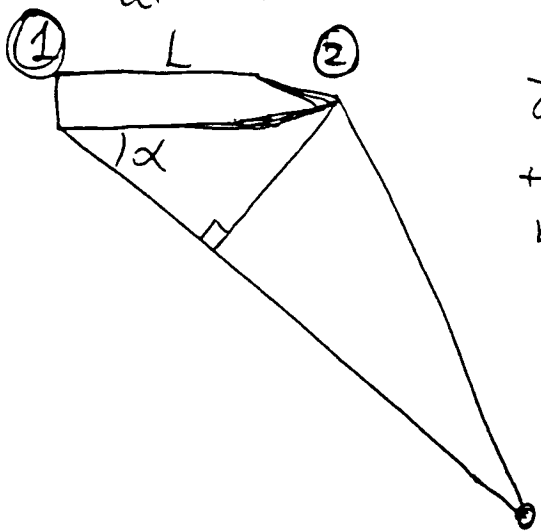
(-1) (2)

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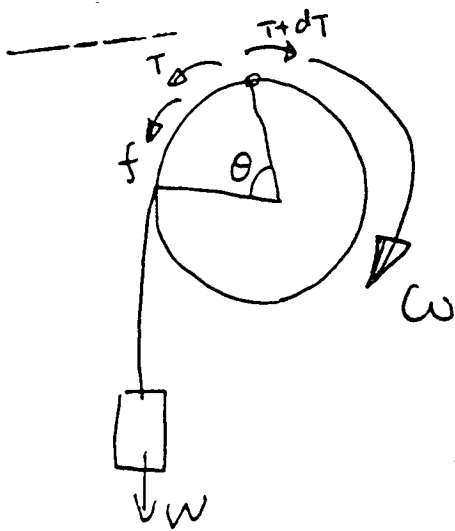
Solution: the key point is to recognize that when one photographs something, he must receive all the photons coming from the object at the same time. +3



from a distant point, the photon from 1 must be emitted $\Delta t = \frac{L \cos \alpha}{c}$ before the photon from 2

\Rightarrow hence the event can be recorded as $(L, \frac{L \cos \alpha}{c})$ in the camera frame for end 2

~~$L = \gamma(L_0 + vt')$
 $L = \gamma(L_0 + v \frac{L \cos \alpha}{c})$
 $L = \gamma L_0 (1 + \frac{v \cos \alpha}{c})$
 $L_0 = \frac{L}{\gamma (1 + \frac{v \cos \alpha}{c})}$~~



Since the positive sense of θ is the same as the angular velocity,

$$T(\theta+d\theta) - T_1(\theta) = df = \mu_k dN(\theta)$$

$$dN(\theta) = T_1(\theta) d\theta$$

friction force $\Rightarrow \frac{dT}{d\theta} = \mu_k T$

on both sides

$$\Rightarrow T(\theta) = W e^{+\mu_k \theta}$$

positive θ direction; $\therefore dT = -df$.

Hence: if choose $T(\theta=0) = W$

the power index in the exponential

should be positive!

Actually, the sailors used to use this kind of mechanism to attain a large frictional force at the other end while applying a rather small force at one end by winding the rope several times around the handle of the rudder.

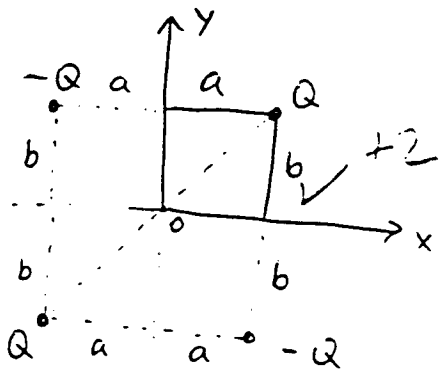
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Solution: use image charges as shown:



hence: a)
$$\underline{f} = \frac{-Q^2}{(2a)^2} \hat{e}_x - \frac{Q^2}{(2b)^2} \hat{e}_y + \frac{Q^2}{4(a^2+b^2)} \frac{+2a\hat{e}_x + 2b\hat{e}_y}{\sqrt{(2a)^2 + (2b)^2}}$$

or:
$$\underline{f} = \left[-\frac{Q^2}{4a^2} + \frac{Q^2 a}{4(a^2+b^2)^{3/2}} \right] \hat{e}_x + \left[-\frac{Q^2}{4b^2} + \frac{Q^2 b}{4(a^2+b^2)^{3/2}} \right] \hat{e}_y$$

b) the electric field is:

$$\underline{E} = \frac{Q [(x-a)\hat{e}_x + (y-b)\hat{e}_y + z\hat{e}_z]}{[(x-a)^2 + (y-b)^2 + z^2]^{3/2}} - \frac{Q [(x-a)\hat{e}_x + (y+b)\hat{e}_y + z\hat{e}_z]}{[(x-a)^2 + (y+b)^2 + z^2]^{3/2}} - \frac{Q [(x+a)\hat{e}_x + (y-b)\hat{e}_y + z\hat{e}_z]}{[(x+a)^2 + (y-b)^2 + z^2]^{3/2}} + \frac{Q [(x+a)\hat{e}_x + (y+b)\hat{e}_y + z\hat{e}_z]}{[(x+a)^2 + (y+b)^2 + z^2]^{3/2}}$$

∴ $\sigma = \frac{1}{4\pi} \nabla \cdot \underline{E} \Rightarrow$ on the plate $\parallel y$,

$$\sigma(x, y, z) = \frac{1}{4\pi} \hat{e}_x \cdot \underline{E}(x, y, z) = \frac{Q}{4\pi} \left[\frac{a}{(a^2 + (y-b)^2 + z^2)^{3/2}} + \frac{a}{(a^2 + (y+b)^2 + z^2)^{3/2}} - \frac{a}{(a^2 + (y-b)^2 + z^2)^{3/2}} + \frac{a}{(a^2 + (y+b)^2 + z^2)^{3/2}} \right]$$

$$= \frac{Q}{2\pi} \left[-\frac{a}{(a^2 + (y-b)^2 + z^2)^{3/2}} + \frac{a}{(a^2 + (y+b)^2 + z^2)^{3/2}} \right]$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

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on the plate $|| x,$

$$\begin{aligned} \sigma(x, 0, z) &= \frac{1}{4\pi} \vec{e}_y \cdot \underline{E}(x, 0, z) = \frac{Q}{4\pi} \left[\frac{-b}{[(x-a)^2 + b^2 + z^2]^{3/2}} + \frac{-b}{[(x-a)^2 + b^2 + z^2]^{3/2}} \right. \\ &\quad \left. + \frac{b}{[(x+a)^2 + b^2 + z^2]^{3/2}} + \frac{b}{[(x+a)^2 + b^2 + z^2]^{3/2}} \right] \\ &= \frac{Q}{2\pi} \left[\frac{b}{[(x+a)^2 + b^2 + z^2]^{3/2}} - \frac{b}{[(x-a)^2 + b^2 + z^2]^{3/2}} \right] \end{aligned}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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the Problem No. 4

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Solution: a) $\nabla^2 \underline{E} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \underline{E}$

& $\underline{E} = \hat{x} E(y) e^{i(k_z z - \omega t)}$

$\Rightarrow \frac{d^2 E(y)}{dy^2} - k_z^2 E(y) = -\frac{\omega^2}{c^2} E(y)$

$\Rightarrow \frac{d^2 E(y)}{dy^2} + (\frac{\omega^2}{c^2} - k_z^2) E(y) = 0$ let $k^2 = \frac{\omega^2}{c^2} - k_z^2$

$\Rightarrow E(y) \propto \sin k y$ & since at $y=0, a$, $E_x = 0$

$\Rightarrow ka = n\pi, n=1, 2, \dots$

hence: $E(y) = E_0 \sin \frac{n\pi y}{a}$

& $\underline{E} = \hat{x} E_0 \sin \frac{n\pi y}{a} e^{i(k_z z - \omega t)}$ & $\frac{\omega^2}{c^2} - k_z^2 = \frac{n^2 \pi^2}{a^2}$

b) $\omega = \frac{5\pi c}{2a}, k_z^2 = \frac{\omega^2}{c^2} - \frac{n^2 \pi^2}{a^2} = \frac{25\pi^2 c^2}{4a^2 c^2} - \frac{n^2 \pi^2}{a^2}$
 $= (\frac{25}{4} - n^2) \frac{\pi^2}{a^2} > 0$ for propagation

$n=1, 2$ hence: $k_{z1} = \frac{\sqrt{21}\pi}{2a}, k_{z2} = \frac{3\pi}{2a}$

c) $\nabla \times \underline{E} = -\frac{1}{c} \frac{\partial \underline{H}}{\partial t} = \frac{i\omega}{c} \underline{H}$

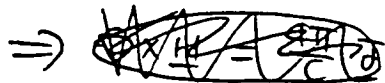
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$$\underline{H} = \frac{c}{i\omega} \nabla \times \underline{E}$$

$$= \frac{c}{i\omega} E_0 \left[ik \sin \frac{n\pi y}{a} e^{i(k_2 z - \omega t)} \hat{y} + \frac{n\pi}{a} \cos \frac{n\pi y}{a} e^{i(k_2 z - \omega t)} \hat{z} \right]$$

∴ at $y=0, a$,

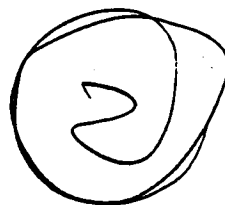
$$\underline{H} = \frac{icE_0}{\omega} \frac{n\pi}{a} \cos \frac{n\pi y}{a} e^{i(k_2 z - \omega t)} \hat{z}$$

surface current \underline{j} :

$$\underline{j}|_{y=0} = \hat{n} \times \underline{H} = -\hat{e}_y \times \underline{H} = \frac{icE_0}{\omega} \frac{n\pi}{a} e^{i(k_2 z - \omega t)} \hat{x}$$

$$\underline{j}|_{y=a} = \hat{n}' \times \underline{H} = +\hat{e}_y \times \underline{H} = +\frac{icE_0}{\omega} (-1)^n \frac{n\pi}{a} e^{i(k_2 z - \omega t)} \hat{x}$$

where: $k_z^2 = \frac{\omega^2}{c^2} - \frac{n^2 \pi^2}{a^2}$



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Solution: i) $Z = \sum_{\epsilon} e^{-\beta \epsilon} = e^{-\beta \epsilon} + e^{-\beta(\epsilon+\Delta)}$

$$F = -kT \ln Z = -kT \ln [e^{-\beta \epsilon} (1 + e^{-\beta \Delta})]$$

where $\beta = \frac{1}{kT}$

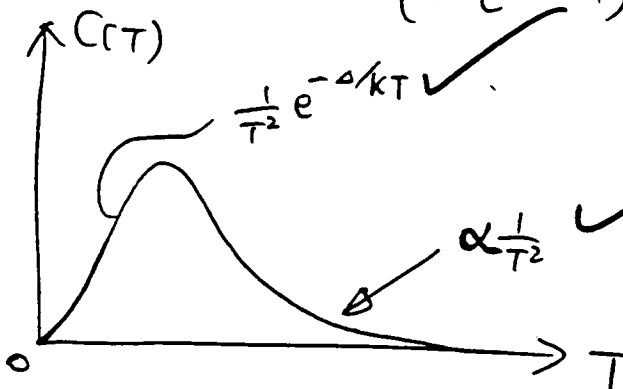
ii) $E = - \frac{\partial \ln Z}{\partial \beta} = \frac{\epsilon e^{-\beta \epsilon} + (\epsilon + \Delta) e^{-\beta(\epsilon + \Delta)}}{e^{-\beta \epsilon} + e^{-\beta(\epsilon + \Delta)}}$

$$= \frac{\epsilon + \Delta e^{-\beta \Delta}}{1 + e^{-\beta \Delta}} = \epsilon + \frac{\Delta}{e^{\beta \Delta} + 1}$$

$$C(T) = \frac{\partial E}{\partial T} = \frac{\frac{\Delta}{kT^2} e^{-\beta \Delta} (1 + e^{-\beta \Delta}) - \frac{\Delta}{kT^2} (\epsilon + \Delta) e^{-\beta \Delta}}{(1 + e^{-\beta \Delta})^2}$$

$$= \frac{(\epsilon + \Delta) \Delta}{kT^2} \frac{e^{-\beta \Delta} (1 + e^{-\beta \Delta}) - \Delta (\epsilon + \Delta) e^{-\beta \Delta}}{(1 + e^{-\beta \Delta})^2}$$

$$= \frac{\Delta^2}{kT^2} \frac{e^{-\Delta/kT}}{(1 + e^{-\Delta/kT})^2} = \frac{\Delta^2}{kT^2} \frac{1}{(1 + e^{\Delta/kT})^2}$$



Note: If you use additional sheets for this problem, number the pages and staple them together.

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iii) from ii), we see

$$C(T) = \frac{\Delta^2}{kT^2} \frac{e^{\Delta/kT}}{(1 + e^{\Delta/kT})^2}$$

take $P(\Delta)$ as broad & flat,
we know as $\Delta/kT \ll 1$ or at low
temperatures,

$C(T)$ goes like $\frac{1}{T^2} e^{-\Delta/kT}$!

0

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution: d) after new equilibrium is reached:

$$Mgh = c \Delta T \Rightarrow \Delta T = \frac{Mgh}{c}$$

(the gravitational potential goes into the heat due to collisions between the ball & the gas molecules)

b) $T ds = dq = c dt$

⇒ $\Delta S = \int \frac{c dt}{T} = c \ln \frac{T + \Delta T}{T} = c \ln \left(1 + \frac{Mgh}{cT} \right)$

c) $p = e^{-\Delta S/k} = e^{-\frac{c}{k} \ln \left(1 + \frac{Mgh}{cT} \right)} = \left(\frac{1}{1 + \frac{Mgh}{cT}} \right)^{c/k}$

d) if $\Delta T/T \ll 1$, $e^{-\Delta S/k} = p = e^{-\frac{c}{k} \ln \left(1 + \frac{\Delta T}{T} \right)} \approx e^{-\frac{c}{k} \cdot \frac{\Delta T}{T}} = e^{-\frac{c}{k} \cdot \frac{Mgh}{cT}} = e^{-Mgh/kT}$

with $mgh = 10^5$ ergs $T = 300$ K
 ⇒ $p \approx e^{-10^5 / (1.38 \times 10^{-23} \times 300)} = e^{-2.5 \times 10^{18}} \approx 10^{-18} \sim 0$

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Solution: $\phi = -E_0 x$ (choose $\phi(x=0) = 0$)

hence $\underline{E} = -\frac{\partial \phi}{\partial x} \hat{e}_x = E_0 \hat{e}_x$

the Hamiltonian is:

$$H = \frac{p^2}{2m} + \frac{1}{2} k x^2 + q\phi$$

$$= -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2} k x^2 - q E_0 x$$

$$= -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2} k \left(x - \frac{q E_0}{k}\right)^2 - \frac{1}{2} k \left(\frac{q E_0}{k}\right)^2$$

$$= -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2} k \left(x - \frac{q E_0}{k}\right)^2 - \frac{1}{2} \frac{q^2 E_0^2}{k} \quad (2)$$

let $x' = x - \frac{q E_0}{k}$ ✓

$$\Rightarrow H(x') = -\frac{\hbar^2}{2m} \frac{d^2}{dx'^2} + \frac{1}{2} k x'^2 - \frac{1}{2} \frac{q^2 E_0^2}{k}$$

∴ this is the Hamiltonian for a normal 1-dimensional harmonic oscillator,

$$(2) E_n = \left(n + \frac{1}{2}\right) \hbar \sqrt{\frac{k}{m}} - \frac{1}{2} \frac{q^2 E_0^2}{k} \quad n=0, 1, 2, \dots$$

∴ if $\phi_n(x)$ is the eigen-function for

$$H(x) = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2} k x^2$$

$$(2) \text{ then } \phi_n(x) = \phi_n\left(x - \frac{q E_0}{k}\right) = e^{-\frac{q E_0}{k} \frac{d}{dx}} \phi_n(x)$$

i.e. just move the equilibrium point. ✓

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

8

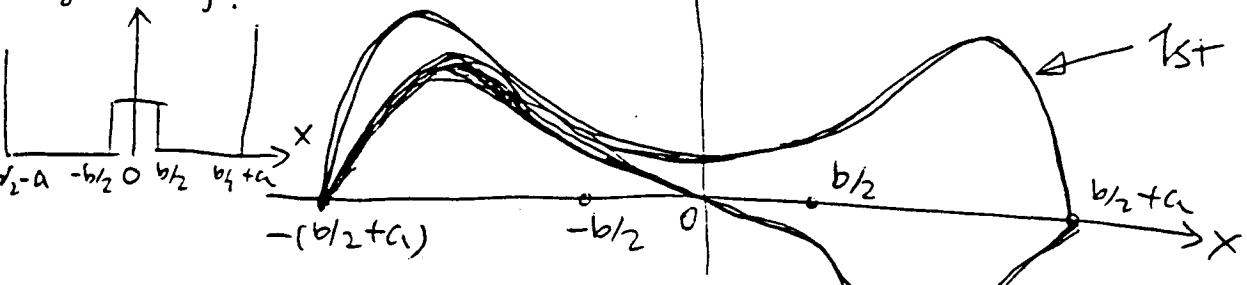
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Solution: a.) the 1st two energy eigenstates look like:

change the origin along x-axis to get symmetry:



1st: symmetric 2nd: anti-symmetric
 barrier penetration!

b.) as $b \rightarrow \infty$, these two eigenstates approach $\frac{1}{\sqrt{2}}(\psi_1 \pm \psi_2)$ respectively, where ψ_1 is the ground wave function of the ~~left~~ well & ψ_2 is that of the ~~right~~ well.

$$E_+ = \frac{\int \psi_+ H \psi_+ dx}{\int \psi_+^2 dx} = \frac{\int (\psi_1 + \psi_2) H (\psi_1 + \psi_2) dx}{\int (\psi_1 + \psi_2)^2 dx}$$

$$E_- = \frac{\int \psi_- H \psi_- dx}{\int \psi_-^2 dx} = \frac{\int (\psi_1 - \psi_2) H (\psi_1 - \psi_2) dx}{\int (\psi_1 - \psi_2)^2 dx}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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$$\Rightarrow E_+ = \frac{\int \varphi_1 H \varphi_1 dx + \int \varphi_2 H \varphi_2 dx + \int \varphi_1 H \varphi_2 dx + \int \varphi_2 H \varphi_1 dx}{\int \varphi_1^2 dx + \int \varphi_2^2 dx + 2 \int \varphi_1 \varphi_2 dx}$$

$$E_- = \frac{\int \varphi_1 H \varphi_1 dx + \int \varphi_2 H \varphi_2 dx - \int \varphi_1 H \varphi_2 dx - \int \varphi_2 H \varphi_1 dx}{\int \varphi_1^2 dx + \int \varphi_2^2 dx - 2 \int \varphi_1 \varphi_2 dx}$$

let $V = \int \varphi_1 H \varphi_2 dx = \int \varphi_2 H \varphi_1 dx$

$\oint. C = \int \varphi_1 \varphi_2 dx, E_0 = \int \varphi_1 H \varphi_1 dx = \int \varphi_2 H \varphi_2 dx$
 $\int \varphi_1^2 dx = \int \varphi_2^2 dx = 1$

$$\Rightarrow E_+ = \frac{2E_0 + 2V}{2(1+C)} = \frac{E_0 + V}{1+C}, E_- = \frac{E_0 - V}{1-C}$$

~~$E_+ = \dots$~~ $\Rightarrow \Delta E_+ = \frac{V - E_0 C}{1+C}, \Delta E_- = \frac{-V + E_0 C}{1-C}$

as $b \rightarrow \infty, C \rightarrow 0, V \rightarrow 0$

hence the 1st symmetric state approach E_0 from below, & the 2nd anti-symmetric state approach E_0 from above.

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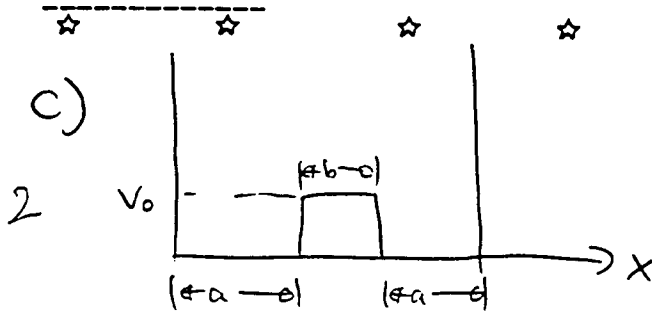
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the eigenstates of ∞ -deep well are:

$$E_n^0 = \frac{n^2 h^2}{8m(2a+b)^2}, n=1,2,\dots$$

$$\psi_n^0 = \sqrt{\frac{2}{2a+b}} \sin \frac{n\pi x}{2a+b}$$

use V_0 as perturbation:

$$H = \begin{cases} H_0 + V_0 \\ H_0 \end{cases}$$

$$a < x < a+b$$

otherwise

$$\Rightarrow E_n = E_n^0 + \int_a^{a+b} V_0 |\psi_n^{(0)}|^2 dx = \frac{n^2 h^2}{8m(2a+b)^2} + \frac{2V_0}{2a+b} \left[\frac{2a+b}{2} \left(\frac{2a+b}{2n\pi} \right) - \frac{2a+b}{4n\pi} \left(\sin \frac{2n\pi(a+b)}{2a+b} - \sin \frac{2n\pi a}{2a+b} \right) \right]$$

$$\psi_n = \psi_n^0 + \sum_m \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0}$$

$$\langle \psi_m^{(0)} | H' | \psi_n^0 \rangle = \frac{2V_0}{2a+b} \int_a^{a+b} \sin \frac{m\pi x}{2a+b} \sin \frac{n\pi x}{2a+b} dx$$

$$= \frac{2V_0}{2a+b} \frac{1}{2} \left[\frac{2a+b}{(m-n)\pi} \left(\sin \frac{(m-n)\pi(a+b)}{2a+b} - \sin \frac{(m-n)\pi a}{2a+b} \right) - \frac{2a+b}{(m+n)\pi} \left(\sin \frac{(m+n)\pi(a+b)}{2a+b} - \sin \frac{(m+n)\pi a}{2a+b} \right) \right]$$

(3)

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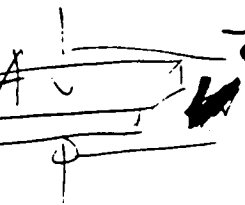
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Solution: 1.) when an electric field \underline{E} is present in the solid, the resulting current density \underline{J} is a function of \underline{E} , & usually $\underline{J} = \sigma \underline{E}$ where σ is the conductivity of the solid, & its inverse $\frac{1}{\sigma} = \rho$ is the resistivity.



we apply a voltage V to a the slab, measure the resulting current I & get: $\rho = \frac{V}{\epsilon I/A}$

2.) when a magnetic field \underline{H} is present, the atoms inside the solid would be magnetized, &

the resulting magnetization \underline{M} usually has the form: $\underline{M} = \chi \underline{H}$ & χ is the magnetic susceptibility of the solid.

we apply an external field \underline{H} to the solid, then turn it off quickly, measure the resulting magnetic field of the solid, get χ from $\underline{M} = \chi \underline{H}$.

1

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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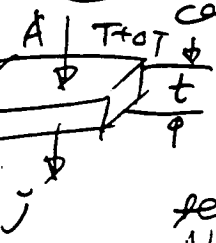
C. when the solid is heated, its temperature would increase. & $dQ = CdT$ holds, where dQ is the heat absorbed, dT the temperature increase & C the heat capacity.

$\frac{C}{M}$ is called specific heat where M is the total mass of the solid.

from above, we know if we measure the $\Delta Q, \Delta T$ then because C is const. at normal temperature, $C = \frac{\Delta Q}{\Delta T} \Rightarrow \frac{C}{m}$ the specific heat.

D. when an inhomogeneous temperature distribution is present, heat flux will result in.

$\vec{j} = -K \nabla T$ where K is called the thermal conductivity. We just use a thin slab, maintain a temperature difference, measure the heat emerging from the lower temperature surface, we get:



$K = \frac{j}{(\Delta T/t)} = \frac{\Delta Q/A}{(\Delta T/t)}$

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heat emerging. -1

2

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution: $L = \frac{1}{2} m v^2 - \frac{e v \cdot A}{c}$

$\Rightarrow H = (m v \cdot \frac{eA}{c})^2 \frac{1}{2m}$

for $\underline{B} = \frac{B_0 x}{L} \hat{z}$ $\nabla \times \underline{A} = \underline{B}$

$\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = \frac{B_0 x}{L}$ let $A_x = A_z = 0$
 $\Rightarrow A_y = \frac{1}{2} \frac{B_0 x^2}{L}$

$\Rightarrow H = \frac{1}{2m} [(m v_x)^2 + (m v_y \cdot \frac{e B_0 x^2}{2c})^2 + (m v_z)^2]$

b) since no y, z dependence,

$m v_y \cdot \frac{e B_0 x^2}{2c} = \text{const.}$

$m v_z = \text{const.}$

also, kinetic energy conserved

$\frac{1}{2} m (v_x^2 + v_y^2 + v_z^2) = \text{const.}$

c) $v_0^2 = v_x^2 + v_y^2$, $v_z = 0$

$v_y = + \frac{e B_0 x^2}{2m c} \Rightarrow \frac{dy}{dx} = \frac{v_y}{v_x} = \frac{e B_0 x^2}{2m c \sqrt{v_0^2 - \frac{e^2 B_0^2 x^4}{4m^2 c^2}}}$

$\Rightarrow Y(x) = \frac{e B_0}{2m c} \int_0^x \frac{x^2 dx}{\sqrt{v_0^2 - \frac{e^2 B_0^2 x^4}{4m^2 c^2}}}$ $Z(x) = 0 = \text{const.}$

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(1)

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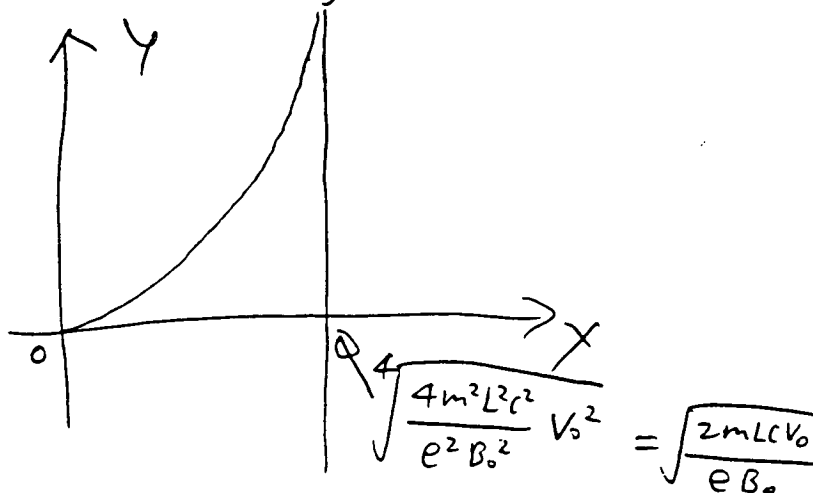
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the trajectory looks like.



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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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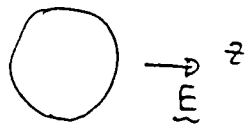
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Solution: d) since $ka \ll 1$, take $\underline{E} = E_0 \hat{e}_z e^{-i\omega t + ik_y y}$
 as $\underline{E} = E_0 \hat{e}_z e^{-i\omega t}$ across the sphere.



$$\nabla^2 \phi = 0 \Rightarrow \phi = \begin{cases} -E r \cos \theta + \frac{d}{r^2} \cos \theta & r > a \\ 0 & r < a \end{cases}$$

ϕ is continuous at $r = a$

$\Rightarrow d = E a^3$ which is the induced dipole moment of the conducting sphere.

show (1) hence: $\underline{d} = a^3 E_0 \hat{e}_z e^{-i\omega t}$

$$e.g. \frac{dP}{d\Omega} = \frac{1}{4\pi c^3} |\underline{d}|^2 \sin^2 \theta = \frac{E_0^2 \omega^4 a^6}{4\pi c^3} \sin^2 \theta$$

is the radiate power, θ is the angle between \hat{n} & \underline{d} .

$$b) \sigma_{scat} = \int \frac{dP/d\Omega}{\frac{c}{4\pi} E_0^2} d\Omega = \frac{E_0^2 \omega^4 a^6}{c^4 E_0^2} \int \sin^2 \theta d\theta d\phi = \frac{8\pi}{3} \frac{\omega^4 a^6}{c^4} = \frac{8\pi}{3} k^4 a^6$$

$$e.g. dI = -\sigma_{scat} I n_0 dl \Rightarrow I = I_0 e^{-n_0 \sigma_{scat} l} = I_0 e^{-l/l_{ext}}$$

where $l_{ext} = \frac{1}{n_0 \sigma_{scat}} = \frac{1}{n_0 \frac{8\pi}{3} k^4 a^6}$ is the extinction length

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution: a.) let \underline{v} be the velocity of the ptcl.

$$\nabla \cdot (\epsilon \nabla \phi) = -4\pi q \delta(x-vt) \quad \phi(k, \omega) = \int \phi e^{i(\underline{k} \cdot \underline{x} - \omega t)} d^3x dt$$

$$\Rightarrow \epsilon(\omega) k^2 \phi(k, \omega) = +4\pi q e^{+i\underline{k} \cdot \underline{v} t}$$

$$\text{hence: } \phi(k, \omega) = \frac{4\pi q}{\epsilon(\omega) k^2} e^{+i\underline{k} \cdot \underline{v} t}$$

$$b.) \underline{E} = -\nabla \phi \Rightarrow \underline{E}_\omega = -i\underline{k} \frac{4\pi q}{\epsilon(\omega) k^2} e^{+i\underline{k} \cdot \underline{v} t}$$

$$\underline{J} = q \underline{v} \delta(x-vt) \Rightarrow \underline{J}_{k, \omega} = q \underline{v} e^{+i\underline{k} \cdot \underline{v} t}$$

$$q. \frac{dQ}{dt} = \int \underline{J} \cdot \underline{E} d^3x = \frac{1}{(2\pi)^3} \int -i\underline{k} \cdot \underline{v} \frac{q^2 4\pi}{k^2 \epsilon(\omega)} e^{-i\omega t} d^3k$$

$$= \frac{1}{(2\pi)^3} \int -i\underline{k} \cdot \underline{v} \frac{q^2 4\pi}{k^2 \epsilon(\omega)} d^3k e^{+i\underline{k} \cdot \underline{v} t}$$

$$q. \frac{dQ}{dt} = \int \text{Re} \left[-i \underline{k} \cdot \underline{v} \frac{q^2 4\pi}{k^2 \epsilon(\omega)} d^3k \right] \frac{1}{(2\pi)^3} e^{i\underline{k} \cdot \underline{v} t}$$

$$= \frac{1}{(2\pi)^3} \int \frac{4\pi q^2}{k^2} \text{Im} \frac{1}{\epsilon(\omega)} \underline{k} \cdot \underline{v} d^3k e^{i\underline{k} \cdot \underline{v} t}$$

is the energy loss rate.

$$\frac{dQ}{dt} = \frac{1}{2\pi} \int d\omega \int \frac{q^2}{k^2} \text{Im} \frac{1}{\epsilon(\omega)} \underline{k} \cdot \underline{v} e^{i\underline{k} \cdot \underline{v} t} d^3k$$

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution: a.) $\Phi = \frac{V^N}{h^{3N}} \int d^3p_1 \int d^3p_2 \dots \int d^3p_N$
 is the phase space volume

$$\Rightarrow P(p_1) d^3p_1 \propto \frac{d\Phi}{d^3p_1} d^3p_1$$

$$\text{or } P(p_1) \propto \int d^3p_2 \dots \int d^3p_N$$

$$\& \frac{p_2^2 + p_3^2 + \dots + p_N^2}{2m} = E - E_1$$

$$\Rightarrow P(p_1) \propto [2m(E - E_1)]^{(N-1)3/2}$$

is the probability

b.) when $N \rightarrow \infty$, $E = N \langle E_i \rangle$

$$\Rightarrow P(p_1) \propto (E - E_1)^{3N/2} \propto \left(1 - \frac{E_1}{N \langle E_i \rangle}\right)^{3N/2}$$

$$\propto e^{-\frac{3E_1}{2 \langle E_i \rangle}}$$

$$\text{i.e. } P(p_1) \propto e^{-\frac{3}{2} N E_1 / E}$$

since: classical Boltzmann distribution

$$\rho \propto e^{-E_i / kT} \Rightarrow \frac{3}{2} \frac{N}{E} = \frac{1}{kT}$$

$$\Rightarrow T = \frac{2}{3} \frac{E}{Nk}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution: a.) $F = -kT \ln Z$ $\mu = \left(\frac{\partial F}{\partial N}\right)_{T,V}$

$$Z = \frac{V^N}{N! h^{3N}} \left[\int e^{-p^2/2mkT} d^3p \right]^N = \frac{1}{N!} \left[\frac{V}{h^3} (2m\pi kT)^{3/2} \right]^N$$

$$\Rightarrow F = -NkT \ln \left[\frac{V}{h^3} (2m\pi kT)^{3/2} \right] + kT \ln N!$$

$$\ln N! \approx N \ln N - N$$

$$\mu = \left(\frac{\partial F}{\partial N}\right)_{T,V} = -kT \ln \left[\frac{V}{h^3} (2m\pi kT)^{3/2} \right] + kT \ln N -$$

$$= +kT \ln \left[\frac{N}{V} \left(\frac{h^2}{2m\pi kT} \right)^{3/2} \right]$$

if use $\bar{p} = \frac{N}{V} kT$

then: $\mu = kT \ln \left[\frac{\bar{p}}{kT} \left(\frac{h^2}{2m\pi kT} \right)^{3/2} \right]$

is the chemical potential for free gas.

b.) $Z_s = \frac{A^N}{N! h^{2N}} \left[\int e^{-p^2/2mkT + \beta \epsilon_0} d^2p \right]^N = \frac{1}{N!} \left[\frac{A}{h^2} (2m\pi kT) e^{\beta \epsilon_0} \right]^N$

$$\Rightarrow F = -kT \ln Z_s = -NkT \ln \left[\frac{A}{h^2} (2m\pi kT) e^{\beta \epsilon_0} \right] + kT \ln N!$$

$$\mu = \left(\frac{\partial F}{\partial N}\right)_{T,V} = -kT \ln \left[\frac{A}{h^2} (2m\pi kT) e^{\beta \epsilon_0} \right] + kT \ln N$$

$$= kT \ln \left[\frac{N}{A} \frac{h^2}{2m\pi kT} e^{-\beta \epsilon_0} \right]$$

is the chemical potential for adsorbed atoms, where A is the area of the plane.

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c) $\mu_g = \mu_s$

$$\Rightarrow kT \ln \frac{\bar{P}}{kT} \left(\frac{h^2}{2\pi m kT} \right)^{3/2} = kT \ln \left[\frac{N}{A} \frac{h^2}{2\pi m kT} e^{-\beta \epsilon_0} \right]$$

$$\Rightarrow \frac{N}{A} = \frac{\bar{P}}{kT} \left(\frac{h^2}{2\pi m kT} \right)^{3/2} e^{\beta \epsilon_0} = \bar{n}_A$$

is the mean # of adsorbed atoms
per unit area. ✓

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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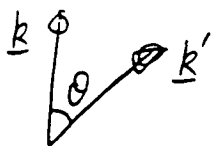
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Solution: use Born approximation,

$$f = -\frac{2m}{\hbar^2} \int e^{-i\mathbf{k}' \cdot \mathbf{r}} \chi(\mathbf{r}) V e^{i\mathbf{k} \cdot \mathbf{r}} \chi(\mathbf{r}) d^3r$$

Q. $\mathbf{p} = \hbar \mathbf{k}$, $\mathbf{p}' = \hbar \mathbf{k}'$

for let $\mathbf{p} \parallel \hat{z}$, $\Rightarrow \chi(\mathbf{r}) = \begin{pmatrix} a \\ 0 \end{pmatrix}$ ~~$\begin{pmatrix} a \cos \theta \\ a \sin \theta \end{pmatrix}$~~



$$\underline{\sigma} \cdot \underline{n} = \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix} + \begin{pmatrix} 0 & -i \sin \theta \sin \phi \\ i \sin \theta \sin \phi & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} \cos \theta & 0 \\ 0 & -\cos \theta \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix}$$

$$(\underline{\sigma} \cdot \underline{n} - \lambda) \begin{pmatrix} a \\ b \end{pmatrix} = 0 \Rightarrow \begin{vmatrix} \cos \theta - \lambda & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \text{if } \lambda = -1 \quad \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sin \frac{\theta}{2} e^{-i\frac{\phi}{2}} \\ \cos \frac{\theta}{2} e^{i\frac{\phi}{2}} \end{pmatrix}$$

$$\Rightarrow \chi(\mathbf{r}') = \begin{pmatrix} \sin \frac{\theta}{2} e^{-i\frac{\phi}{2}} \\ \cos \frac{\theta}{2} e^{i\frac{\phi}{2}} \end{pmatrix}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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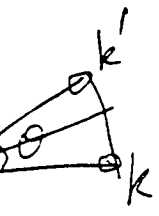
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$$\chi_{(\underline{s})}^\dagger \underline{L} \cdot \underline{S} \chi_{(\underline{s})} = (\sin \frac{\theta}{2} e^{i\frac{\phi}{2}}, \cos \frac{\theta}{2} e^{-i\frac{\phi}{2}}) \underline{L} \cdot \underline{S} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\underline{L} \cdot \underline{S} = \frac{\hbar}{2} \left[\begin{pmatrix} l_x & 0 \\ 0 & l_x \end{pmatrix} + \begin{pmatrix} 0 & -i l_y \\ i l_y & 0 \end{pmatrix} + \begin{pmatrix} l_z & 0 \\ 0 & -l_z \end{pmatrix} \right]$$

$$= \frac{\hbar}{2} \begin{pmatrix} l_z & l_x - i l_y \\ l_x + i l_y & -l_z \end{pmatrix} \quad l_{\pm} = l_x \pm i l_y$$



$$\Rightarrow \chi_{(\underline{s})}^\dagger \underline{L} \cdot \underline{S} \chi_{(\underline{s})} = \frac{\hbar}{2} (\sin \frac{\theta}{2} e^{i\frac{\phi}{2}}, \cos \frac{\theta}{2} e^{-i\frac{\phi}{2}}) \begin{pmatrix} l_+ & l_- \\ l_- & -l_+ \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \frac{\hbar}{2} (\sin \frac{\theta}{2} e^{i\frac{\phi}{2}} l_+ + \cos \frac{\theta}{2} e^{-i\frac{\phi}{2}} l_-)$$

$$= -\frac{m}{2\pi\hbar^2} \frac{\hbar}{2} \int e^{-i\mathbf{k}' \cdot \mathbf{r}} (\sin \frac{\theta}{2} e^{i\frac{\phi}{2}} l_+ + \cos \frac{\theta}{2} e^{-i\frac{\phi}{2}} l_-) \Theta\left(\frac{e^{i\mathbf{k}' \cdot \mathbf{r}}}{r}\right) d^3\mathbf{r}$$

$$= -\frac{m}{4\pi\hbar} \int e^{-i\mathbf{k}' \cdot \mathbf{r}} \frac{e^{-\mu r}}{r} \cos \frac{\theta}{2} e^{-i\frac{\phi}{2}} l_- \Sigma i^l (2l+1) P_l(\cos \theta) j_l(kr) d^3\mathbf{r}$$

Since $P_l(\cos \theta) = \sqrt{\frac{4\pi}{2l+1}} Y_{l,0}$, $l_z e^{i\mathbf{k}' \cdot \mathbf{r}} = 0$

$$l_+ Y_{l,0} = \sqrt{l(l+1)} Y_{l,1}$$

$$\Rightarrow f = -\frac{m}{4\pi\hbar} \int e^{-i\mathbf{k}' \cdot \mathbf{r}} \frac{e^{-\mu r}}{r} \cos \frac{\theta}{2} e^{-i\frac{\phi}{2}} \frac{\sqrt{4\pi}}{2} (2l+1) \sqrt{l(l+1)} Y_{l,1} j_l(kr) d^3\mathbf{r}$$

$$= -\frac{m \cos \frac{\theta}{2} e^{-i\frac{\phi}{2}}}{4\pi\hbar} \int \frac{e^{-\mu r}}{r} e^{-i\mathbf{k}' \cdot \mathbf{r}} \Sigma j_l(kr) i^l \frac{\sqrt{4\pi}}{2} (2l+1) \sqrt{l(l+1)} Y_{l,1} d^3\mathbf{r}$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

(2)

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$$\Rightarrow f \propto \omega^{\frac{\theta}{2}} e^{+i\frac{\varphi}{2}} g(\theta, \varphi)$$

$$\frac{d\sigma}{d\Omega} = |f|^2 \propto \omega^{\frac{2\theta}{2}} g^2(\theta, \varphi)$$

3

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution: a)
$$P = \frac{1}{\hbar^2} \left| \int_0^t \gamma e^{-i\omega t} e^{i\Delta E t / \hbar} dt \right|^2$$

$$= \frac{\gamma^2}{\hbar^2} \left| \frac{e^{i(\omega + \Delta E / \hbar)t} - 1}{-i(\omega + \Delta E / \hbar)} \right|^2$$

$$= \frac{\gamma^2}{\hbar^2} \frac{2 - 2\cos(\omega + \Delta E / \hbar)t}{(\omega + \Delta E / \hbar)^2} = \frac{4\gamma^2 \sin^2\left(\frac{(\omega + \Delta E / \hbar)t}{2}\right)}{(\omega + \Delta E / \hbar)^2}$$

b) $H\psi = i\hbar \frac{\partial}{\partial t} \psi$

$$H = \begin{pmatrix} 0 & \gamma e^{i\omega t} \\ \gamma e^{-i\omega t} & \Delta E \end{pmatrix}$$

$$\psi = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} 0 & \gamma e^{i\omega t} \\ \gamma e^{-i\omega t} & \Delta E \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = i\hbar \begin{pmatrix} \dot{a} \\ \dot{b} \end{pmatrix}$$

$$\Rightarrow \frac{1}{i\hbar} \begin{pmatrix} \gamma b e^{i\omega t} \\ \gamma a e^{-i\omega t} + \Delta E b \end{pmatrix} = \begin{pmatrix} \dot{a} \\ \dot{b} \end{pmatrix}$$

$$\Rightarrow \begin{cases} \frac{da}{dt} = \frac{\gamma}{i\hbar} b e^{i\omega t} \\ \frac{db}{dt} = \frac{\gamma}{i\hbar} a e^{-i\omega t} + \frac{\Delta E}{i\hbar} b \end{cases}$$

~~$\frac{d^2 a}{dt^2} = \frac{\gamma}{i\hbar} \frac{db}{dt} e^{i\omega t}$~~
 ~~$\frac{d^2 b}{dt^2} = \frac{\gamma}{i\hbar} \frac{da}{dt} e^{-i\omega t} + \frac{\Delta E}{i\hbar} \frac{db}{dt}$~~

Let $b = b' e^{-i\omega t} \Rightarrow \frac{db'}{dt} - i\omega b' = \frac{\gamma}{i\hbar} a + \frac{\Delta E}{i\hbar} b'$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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$$q. \begin{cases} \frac{da}{dt} = \frac{\gamma}{i\hbar} b' \\ \frac{db'}{dt} = \frac{\gamma}{i\hbar} a + \left(\frac{\Delta E}{i\hbar} + i\omega \right) b' \end{cases}$$

$$\Rightarrow \frac{d^2 b'}{dt^2} = -\frac{\gamma^2}{\hbar^2} b' - \frac{i}{\hbar} (\Delta E - \hbar\omega) \frac{db'}{dt}$$

$$b' = e^{i\Omega t}$$

$$\Rightarrow \Omega^2 = \frac{\gamma^2}{\hbar^2} + \frac{\Delta E - \hbar\omega}{\hbar} \Omega$$

$$\Rightarrow \Omega_{\pm} = \frac{\Delta E - \hbar\omega}{\hbar} \pm \sqrt{\left(\frac{\Delta E - \hbar\omega}{\hbar} \right)^2 - 4\gamma^2/\hbar^2}$$

$$\Rightarrow b' = A e^{i\Omega_+ t} + B e^{i\Omega_- t} \quad b' = 0 \text{ at } t = 0$$

$$\Rightarrow A = -B \quad b' = A e^{i\Omega_+ t} - e^{i\Omega_- t}$$

$$a = \frac{\gamma}{i\hbar} A \left[\frac{e^{i\Omega_+ t} - 1}{i\Omega_+} - \frac{e^{i\Omega_- t} - 1}{i\Omega_-} \right] \quad a = 0 \text{ at } t = 0 \Rightarrow |A|$$

$$\Rightarrow P = |b'|^2 = |A|^2 \left| e^{i\Omega_+ t} - e^{-i\Omega_- t} \right|^2$$

2

You probably will go to the next part of the problem!

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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with $u(0, x, y, z) = \Phi(x, y, z)$

$$\Rightarrow A_{l,m,n} = \frac{8}{D^3} \int_0^D \int_0^D \int_0^D \Phi(x, y, z) \sin \frac{l\pi x}{D} \sin \frac{m\pi y}{D} \sin \frac{n\pi z}{D} dx dy dz$$

$$= \frac{8}{D^3} \int_0^D dx \int_0^D dy \int_0^D dz \Phi(x, y, z) \sin \frac{l\pi x}{D} \sin \frac{m\pi y}{D} \sin \frac{n\pi z}{D}$$

$l, m, n = 1, 2, 3, \dots$

o.) for exponentially grow, $\beta - \frac{\alpha \pi^2}{D^2} (l^2 + m^2 + n^2) < 0$

$$\Rightarrow \beta > \frac{\alpha \pi^2}{D^2} (3) \Rightarrow D^2 > \frac{3\pi^2 \alpha}{\beta} = D_c^2$$

$$\Rightarrow D_c = \pi \sqrt{\frac{3\alpha}{\beta}}$$

2

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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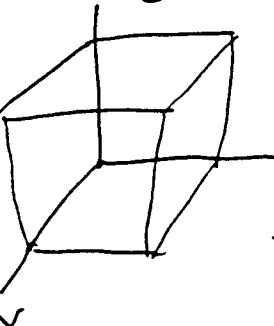
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The Problem No. 19

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Solution: a) $\frac{\partial u}{\partial t} = \alpha \nabla^2 u + \beta u$

let $u = X(x)Y(y)Z(z)T(t)$



$$\Rightarrow \frac{\partial T}{\partial t} / T = \alpha \left(\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} \right) + \beta = \lambda_0^2$$

$$\Rightarrow T \propto e^{\lambda_0^2 t}$$

$$\text{also } \frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = (\lambda_0^2 - \beta) / \alpha$$

$$\text{let } X \propto \sin k_x x \quad Y \propto \sin k_y y \quad Z \propto \sin k_z z$$

$$\Rightarrow k_x^2 + k_y^2 + k_z^2 = -(\lambda_0^2 - \beta) / \alpha$$

g. $u(x, y, z, t) = 0$ at boundary,

$$\Rightarrow k_x = \frac{l\pi}{D} \quad \begin{cases} k_x = \frac{l\pi}{D} \\ k_y = \frac{m\pi}{D} \\ k_z = \frac{n\pi}{D} \end{cases}$$

$$\text{hence: } (l^2 + m^2 + n^2) \frac{\pi^2}{D^2} = \frac{\beta - \lambda_0^2}{\alpha}$$

$$\text{or: } \lambda^2 = \beta - \alpha \frac{\pi^2}{D^2} (l^2 + m^2 + n^2)$$

$$\Rightarrow u(x, y, z, t) = \sum_{l, m, n} A_{lmn} \sin \frac{l\pi x}{D} \sin \frac{m\pi y}{D} \sin \frac{n\pi z}{D} e^{\left[\beta - \frac{\alpha \pi^2}{D^2} (l^2 + m^2 + n^2) \right] t}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution: $T_f = -\frac{BN^2}{2AV_f^2}$ $T_i = -\frac{BN^2}{2AV_i^2}$ $\frac{-BN^2}{2A} = \frac{T_i V_i^2}{1}$

$\rightarrow T_f = \frac{\cancel{T_i V_i^2}}{V_f^2}$

(c) $V_i < V_f$
 $\therefore T_f < T_i$ ✓ 3

for a real system, when a ^{free} ~~static~~ expansion occur,)
~~the temperature~~ the potential energy of the system will
 increase (since the forces are attractive, the larger the distance
 the higher the potential) , So the kinetic energy will
 decrease. So temperature decreases.