UCSD Physics Departmental Written Examination

Fall 1990

Department of Physics

University of California, San Diego

La Jolla, California 92093-0319

Written Departmental Examination - Fall, 1990-91

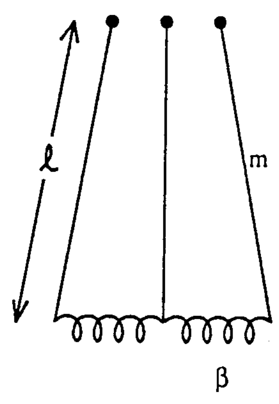
PART I

Instructions

Each problem is worth 10 points. You are to work problems 1-8 plus **either** problem 9 **or** problem 10.

Problem 1

A line of N pendulums, each consisting of a massive rod of mass m and length ℓ, are positioned with their pivots along a line at equal distance from each other. The pivots are at the upper end of each bar, at the lower end the bars are connected by springs with spring constant β. The pendulums are constrained to move in the vertical plane that contains the pivots. Assume periodic boundary conditions, that is assume that the first and last pendulums are coupled together by a spring too.



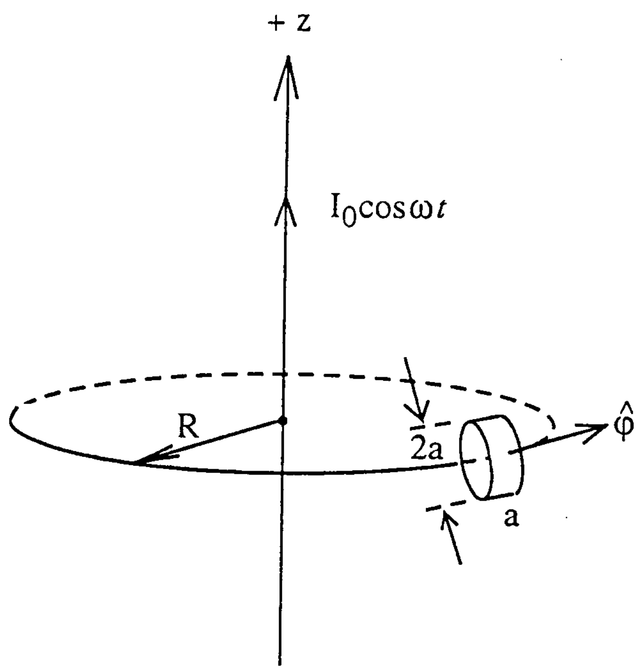
(a) Find the equations of motion.

(b) Find the normal mode frequencies of the system.

(c) Describe the motion that corresponds to the lowest and highest values of the frequency.

Problem 2

A conducting object with permeability $\mu = 1$ (relative to the vacuum value) is placed at a distance R from an infinitely long thin wire that carries an alternating current $I_0 \cos \omega t$. The object is a solid cylinder of radius a and length a, where $a \ll R$; its axis is aligned along the $\hat{\phi}$ direction in a cylindrical coordinate system whose z-axis is along the wire. The cylinder's resistivity η is sufficiently large that the skin depth for fields of frequency ω is much larger than a.



Working in the quasi-static limit (ω small), derive an expression for the thermal power dissipated in the cylinder due to eddy currents.

Problem 3

(a) Explain briefly what is spin-orbit interaction.

(b) Explain briefly the effects of the spin-orbit interaction on the spectrum of atomic hydrogen.

(c) Sketch the energy levels of hydrogen with the principal quantum number $n = 2$ taking into account the spin-orbit interaction. Label each level with its total, orbital and spin angular momenta in the usual notation, and indicate the degeneracy of each level.

(d) Indicate qualitatively the change in the energy levels in (c) when a weak external magnetic field (of about 10 Gauss) is applied.

Problem 4

Consider as a hypothetical model for a star an ideal gas sphere in hydrostatic equilibrium at a uniform temperature T_0. Assume spherical symmetry.

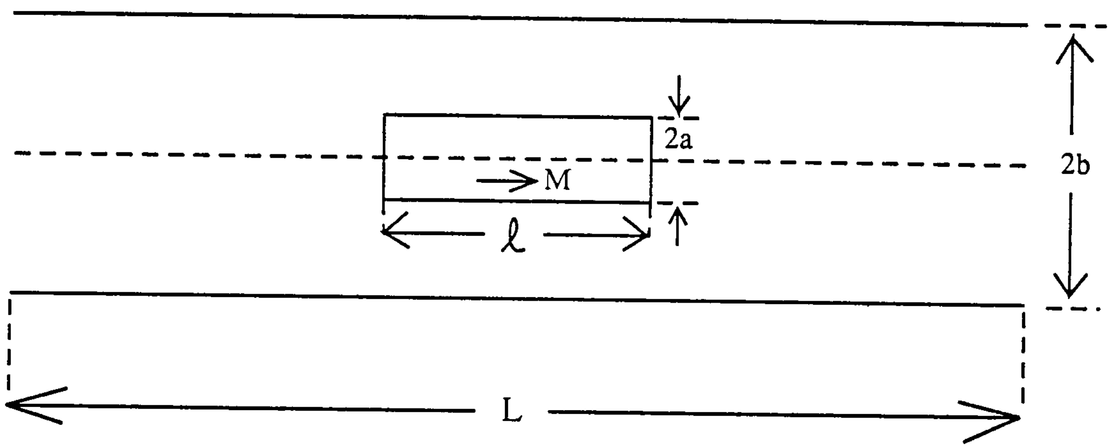
(a) Find an equation for the pressure derivative dP/dr at radius r, in terms of the gravitational constant G, the density $\rho(r)$ at radius r and the mass interior to r, $M(r)$.

(b) Assume the star undergoes a *uniform* contraction: the distance between any two mass elements changes by the same fraction y. Find the change in the star's temperature necessary to maintain hydrostatic equilibrium. You may neglect the effect of radiation pressure.

(c) Assuming the surface of the star emits radiation as a black body, and based on your answer in (b): How will the color of this star change as it contracts? How will the luminosity L change? (L=energy radiated per unit time).

Problem 5

A cylindrical solid rod of radius a and length ℓ ($\ell \gg a$) has a permanent uniform magnetization M parallel to its axis. The rod is located at the center of a thin superconducting cylindrical shell of length L ($L \gg \ell$) and radius b ($L \gg b$), and coaxial with it.



(a) Calculate $\vec{B}$ and $\vec{H}$ near the center of the rod as functions of radius r measured from the axis in the range $0 < r < b$.

(b) Calculate the surface current per unit length in the shell near the center of the rod.

Problem 6

A box is divided into three sections, red, white and blue, as shown in the figure:

red	white	blue

The orthonormal states ψ_r, ψ_w, ψ_b of a quantum mechanical particle form a complete basis set with ψ_r designating the state where the particle is found with certainty in the red section, etc.

(a) R is an operator which moves the particle one section to the right except that at the right end section

$$R\psi_b = 0.$$

L is an operator which moves the particle one section to the left, except that at the left end section

$$L\psi_r = 0.$$

Find the matrix representations of R and L. Are they physical observables?

(b) The Hamiltonian of the particle is given by

$$H = \frac{\hbar\omega}{\sqrt{2}}(R + L)$$

where ω is a (positive) angular frequency. Find the energy eigenvalues of the particle.

(c) The particle is found initially to be in the white section. Calculate the probablity of finding the particle in the red section at time t afterwards.

Problem 7

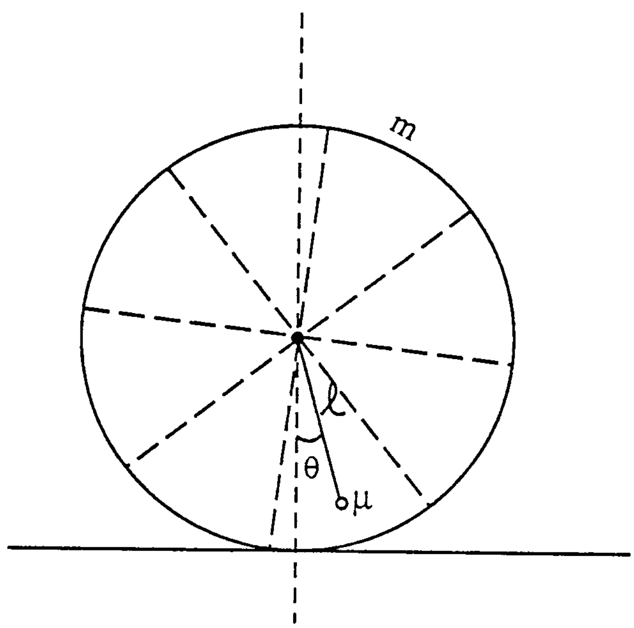
Consider a solid composed of atoms with magnetic moment μ per atom. Assume the moments interact weakly with the lattice vibrations in the solid but not with each other. In an applied magnetic field H a moment has energy $\epsilon = -\mu H$ ($\epsilon = +\mu H$) if it points parallel (antiparallel) to the field, and we assume that no other orientations are possible. The system is thermally insulated, initially in equilibrium at temperature T with applied field H, and the field is suddenly switched from H to $-H$.

(a) Find the new temperature of the system, T', after equilibrium has been reached, in terms of the given quantities and the specific heat per atom c due to lattice vibrations. You may assume $|T' - T| \ll T$. Is T' larger or smaller than T?

(b) Give a numerical estimate as for which initial temperature range the change in temperature in this process will be larger than 1% for $H = 10^5$ gauss. Assume $\mu = \mu_B$ (Bohr magneton). $\mu_B = 0.927 \times 10^{-20}$ ergs/gauss, $k_B = 1.38 \times 10^{-16}$ ergs/K.

Problem 8

A bicycle wheel consisting of a rim with mass m and massless spokes, rolls without slipping on a flat horizontal surface. A pendulum bob of mass μ is suspended from the center of the wheel by a massless rod of length ℓ. Assume that the pendulum is free to swing, all motion takes place in the vertical plane that contains the wheel, the pendulum angle θ between the rod and the vertical direction is small.



(a) Find the equations of motion.

(b) Solve the equations of motion.

(c) Discuss the limiting cases $m \ll \mu$ and $m \gg \mu$.

(d) Describe the relative motion of the wheel and the pendulum.

Problem 9

(a) $J\ (z)$ is a solution of the differential equation $zJ'' + J' = zJ(z) = 0$ with $J(0) = 1$. Give the first two non-zero terms in the series expansion of $J(z)$ about $z = 0$.

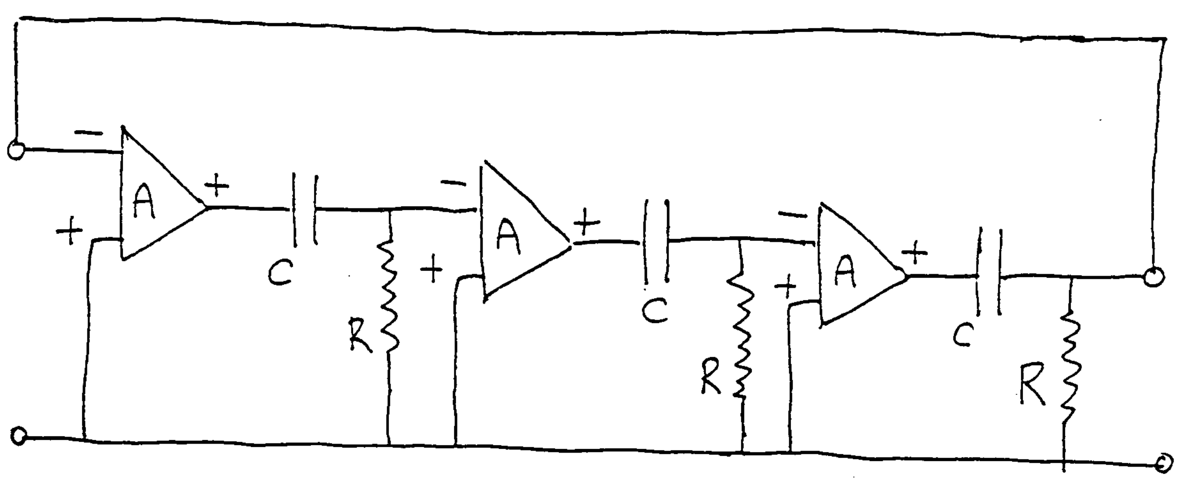
(b) Find the first two terms in the asymptotic expansion of

$$F(t) = \int_0^{\frac{\pi}{2}} \exp\{t\cos\theta\}\theta^2 J(\theta)d\theta$$

as $t \to +\infty$.

Problem 10

Three inverting amplifiers and three CR-circuits are connected as shown in the figure below. Each of the amplifiers has a gain A, infinite input impedance, and zero output impedance.



(a) What is the minimum gain A for which the circuit will oscillate?

(b) What is the frequency at which the circuit will oscillate?

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PART II

Instructions

Each problem is worth 10 points. You are to work problems 11-18 plus **either** problem 19 **or** problem 20.

Problem 11

A particle of mass μ moves in an attractive central force field $F(r) = -k/r^{\beta+1}$, where k and β are constants.

(a) Construct the effective potential $U_{eff}(r)$, for which the radial equation of motion satisfies $\mu\ddot{r} = -\partial U_{eff}/\partial r$.

(b) Sketch $U_{eff}(r)$ for the cases (i) $\beta < 0$, (ii) $2 > \beta > 0$, (iii) $\beta > 2$. For what values of β does a stable circular orbit exist? For what values of β are all orbits bounded?

(c) For those values of β which support a stable circular orbit, calculate the radius, r_0, of the stable circular orbit in terms of the (conserved) angular momentum and other constants.

(d) Let $r = r_0 + \eta$, and derive the equation of motion for radial deviations, $\eta(t)$, assuming η is small. Under what conditions will the perturbed orbit be closed?

Problem 12

A point dipole P is placed at the center of a solid dielectric sphere of radius a and dielectric constant ϵ. Find the electric field at points inside and outside the sphere.

Hint: The field outside is a dipole field; inside another term is needed in addition to a dipole field.

Problem 13

(a) A non-relativistic particle moves in one dimension along the x axis with momentum p in the positive direction. At $x = 0$ there is a very narrow potential wall described by the potential

$$V(x) = \frac{\hbar^2}{2m}\beta \cdot \delta(x)$$

where β is a positive parameter to characterize the strength of the wall and $\delta(x)$ is the Dirac delta function. Calculate the reflection and transmission coefficients.

(b) Calculate the scattering states and the bound state for negative β. Show that the scattering states are orthogonal to the bound state.

(c) What is the interpretation of the pole at imaginary momentum in the scattering amplitude for negative β?

Problem 14

Consider a system of N noninteracting free fermions of mass m and spin 1/2 in a box of volume V at zero temperature. The system is thermally insulated. Assume that suddenly the fermions turn into bosons of spin 0, with no change in their mass. Will the system Bose-condense?

Hints:

$$\int_0^\infty dx \frac{x^{1/2}}{e^x - 1} = 2.32$$
$$\int_0^\infty dx \frac{x^{3/2}}{e^x - 1} = 1.78$$

Density of states per spin for particles of mass m in box of volume V:

$$g(\epsilon) = \frac{V}{4\pi^2}(\frac{2m}{\hbar^2})^{3/2}\epsilon^{1/2}$$

Problem 15

In the ground state of an atom the electrons and the nucleus interact through the Coulomb force (spin related interactions ignored). Assume that the nucleus has infinite mass. Prove the Virial theorem

$$2\overline{E}_{kin} + \overline{E}_{pot} = 0$$

where $\overline{E}_{kin}, \overline{E}_{pot}$ designate the average kinetic and potential energies, respectively, in the ground state.

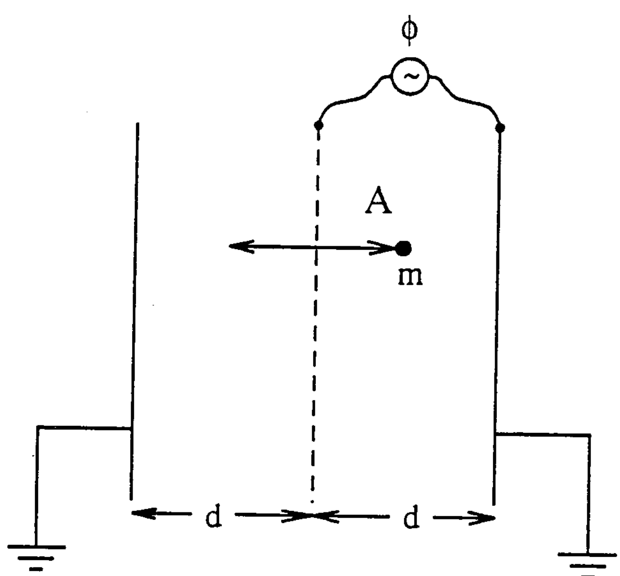
Hint: The potential energy of the atom with N electrons satisfies the scaling law $V(\lambda\vec{r}_1, ..., \lambda\vec{r}_N) = \frac{1}{\lambda}V(\vec{r}_1, ..., \vec{r}_N)$. Apply the variational principle to the one-parameter set of trial wavefunctions

$$\lambda^{\frac{3}{2}N} \cdot \psi_o(\lambda\vec{r}_1, ..., \lambda\vec{r}_N)$$

where $\psi_0(\vec{r}_1, ..., \vec{r}_N)$ is the exact ground state wave function. Note that the trial wavefunctions are normalized.

Problem 16

A grid is located half way between the plates of a parallel plate capacitor (a distance d from each plate). The plates are grounded and the grid is maintained by a voltage generator at the potential $\phi = -V_0$, where $V_0 > 0$. A heavy ion of mass m and charge $+e$ executes oscillations back and forth through the grid; the grid is effectively transparent to the ion. The initial amplitude of oscillation (maximum displacement from the grid) is A_0, where A_0 is large compared to the wire spacing in the grid and yet smaller than d. What is the final amplitude of oscillation after the grid potential has been changed very slowly (compared with the period of oscillation) to $\phi = -2V_0$.



Problem 17

A certain thermodynamic system obeys the equation of state $P = A \cdot N^2 V^{-2} T^3$ and has heat capacity $C_V = BN^4 V^{-3} T$. Initially, the system is maintained in equlibrium at temperature T_i and volume V_i with particle number N. The system is then allowed to expand freely to a volume V_f and subsequently relax to a new equilibrium state (T_f, V_f, N). During the expansion the gas is not in equilibrium, however no work is done by the gas nor is any heat exchanged with its environment.

(a) What is the change in energy of the gas $\Delta E = E_f - E_i$?

(b) Compute T_f in terms of T_i, V_f, V_i, and N.

(c) Is T_f larger, smaller or equal to T_i? Explain whether your answer could (qualitatively) correspond to a real physical system and if you found a change in temperature what its microscopic origin could be.

Problem 18

An electromagnetic wave $\underline{E} = E_o \hat{z} e^{ik.x}$ is incident on an infinite conducting cylinder with axis along $\hat{z}$. The cylinder has radius a.

(a) Calculate the amplitude of the scattered wave.

(b) Derive a general expression for the cross-section.

(c) Discuss your result for (b) in the limit $ka \ll 1$.

Problem 19

Use Fourier transformations to find a particular solution to the partial differential equation:

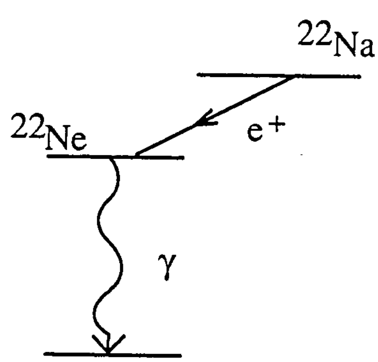
$$\frac{\partial \psi}{\partial t} - \frac{\partial^2 \psi}{\partial x^2} + 8\psi = \delta(x - 2t)$$

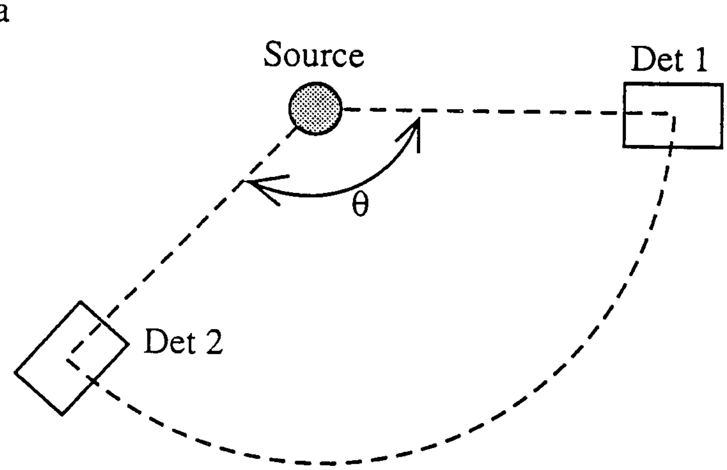
for all x and t.

Problem 20

^{22}Na is a positron and gamma emitter as shown in the figure below. Assume that the positron and gamma are emitted simultaneously. A ^{22}Na source with a strength of 50mCu is surrounded by a thin layer of material in which the positrons are slowed down such that they are captured by atomic electrons in that material and form positronium. The positronium decays into two gamma rays. Gamma rays penetrate the material unobstructed.

Two detectors, one of which is movable along a circle as indicated in the other figure below, are designed to detect gamma rays with an efficiency of 25%. Each detector subtends a solid angle $\Delta\Omega = 10^{-3}$ strad. The detectors are connected with equal length cables to a coincidence circuit (with resolving time $\tau = 50$ ns) whose output is connected to a scaler. The coincidence circuit gives an output only if the two input signals arrive at its input within the time τ. 1 Cu corresponds to 3.7 x 10^{10} disintegrations per second.





(a) Sketch in a figure the coincidence rate as function of the angle θ between the two detectors. Briefly comment on its most important features in different regions of θ.

(b) What are the singles rates in each detector?

(c) What is the coincidence rate measured by the scaler?

(d) What is the accidental coincidence rate (rate due to two distinct disintegrations within the resolving time τ)?

(e) Put some approximate numbers along the vertical axis of your figure in (a).

(f) How would you measure the accidental coincidence rate?

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Score = 6

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the Problem No. 1 and your Identification No. . 59.

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Solution:

(a) 3/1 $I\ddot{\theta}_i = -\beta l^2 (\theta_i - \theta_{i+1}) - \beta l (\theta_i - \theta_{i-1}) - mgl \frac{\theta_i}{q}$

$I = ?$ $i = 1, N$ $\theta_{N+1} = \theta_1$, $\theta_{1-1} = \theta_N$ (periodic boundary)

$I = \frac{1}{3} m l^2$

$\Rightarrow$ $\ddot{\theta}_i \circ + \omega'^2 (2\theta_i - \theta_{i+1} - \theta_{i-1}) + \omega_0^2 \theta_i = 0$

b) 2/4

$\omega_0^2 = \frac{mg\,l/2}{I} = 3g/2l$

$\omega'^2 = \frac{\beta l^2}{I} = 3\beta/m$

(b) normal mode frequencies ω^2 are eigen values of the bel matrix

$$\begin{vmatrix} \omega^2 - 2\omega'^2 - \omega_0^2 & \omega'^2 & & \omega'^2 \\ \omega'^2 & \omega^2 - 2\omega'^2 - \omega_0^2 & \omega'^2 & \\ & & \ddots & \\ \omega'^2 & & \omega'^2 & \omega^2 - 2\omega'^2 - \omega_0^2 \end{vmatrix} = 0$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solve it. $\Rightarrow$ $\omega_1, \cdots \omega_N$.

$\Rightarrow$

(c) $\frac{1}{2}$ $\omega^2 = \omega_0^2$ is the lowest frequency.

Correspond ~~that~~ to the condition that the rods oscillate ~~move~~ tog

with the same phase. no relative motion.

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 10

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the Problem No. 2 and your Identification No. . 59

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Solution:

$$\cancel{P = \rho \int \sigma E^2 \, dV} \qquad \eta = \frac{1}{\sigma}$$

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j}$$

$$\vec{j} = \sigma \vec{E}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t}$$

$$\vec{H} = \frac{4\pi}{2\pi R c} I = \frac{2}{Rc} I \, \vec{e_\varphi}$$

1

$$\nabla \times \vec{E} = + \frac{2}{Rc^2} \omega I_0 \sin\omega t \, \vec{e_\varphi}$$

$$\Rightarrow \cancel{\vec{E} = \frac{2}{Rc^2} \omega I_0 \sin\omega t}$$

$$2\pi r E = \frac{2}{Rc^2} \omega I_0 \sin\omega t \, \pi r^2$$

$$E = \frac{r\omega I_0 \sin\omega t}{Rc^2}$$

1

$$\Rightarrow \cancel{P = \int \vec{j} \cdot \vec{E} \, dV}$$

$$\cancel{= \int \frac{\vec{E}}{\eta} \cdot \vec{E} \, dV}$$

$$\cancel{a \ll R}$$

$$\cancel{\Rightarrow P = \frac{E^2(R)}{\eta} \cdot \pi a^3}$$

$$P = \int \frac{E^2}{\eta} \, dv$$

$$= \frac{a}{\eta} \int E^2 \, 2\pi r \, dr$$

$$= \frac{a}{\eta} \cdot 2\pi \left(\frac{\omega I_0}{Rc^2}\right)^2 \sin^2\omega t \int_0$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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the Problem No. 2 and your Identification No. . 19

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$$= \frac{a^4}{4} \frac{a}{\eta} (2\pi) \left(\frac{\omega I_0}{Rc^2}\right)^2 \sin^2 \omega t$$

$$\overline{P} = \frac{1}{2} |P_{max}| = \frac{a^5}{4\eta} \pi \left(\frac{\omega I_0}{Rc^2}\right)^2 \checkmark$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 8

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the Problem No. 3 and your Identification No. 59

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Solution:

(a) Spin-orbit interaction is the interaction between spin angular momentum and orbital angular momentum. We know angular momentum corresponds to a ~~cer~~ certain magnetic moment (if g factor not zero). So this interaction is a magnetic interaction between two magnetic moments.

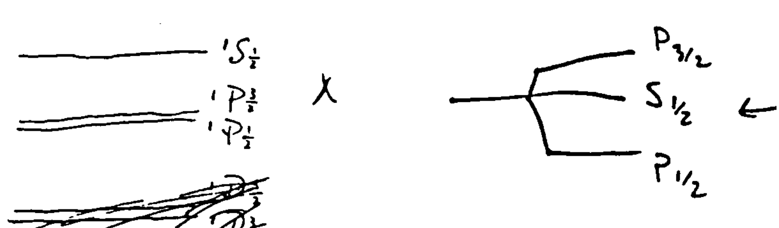
(b) first: Effect of spin-orbit interaction makes ~~the sp~~ the energy levels ~~w~~ with the same spins and same orbital angular momentum but with different total angular momentums to separate. That is, it decreases the ~~de~~ degree of degeneracy of ~~each~~ hydrogen. So ~~the spectrum will change~~ some lines in the spectrum will split into two.

Second this interaction will change energy levels. So lines in the spectrum will change

(c) $n=2$ $S=\frac{1}{2}$, $L=0,1,\cancel{2}$.

$\Rightarrow \ {}^{2S+1}L_J \Rightarrow \ {}^1S_{\frac{1}{2}} \quad {}^1P_{\frac{1}{2},\frac{3}{2}}$

~~ony~~ energy level:



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Degeneracy:

$^1S_{\frac{1}{2}}$	2
$^1P_{\frac{1}{2}}$	2 ✓
$^1P_{\frac{3}{2}}$	4
~~$^1D_{\frac{3}{2}}$~~	~~4~~
~~$^1D_{\frac{5}{2}}$~~	~~6~~

(d) when a weak external magnetic field is applied, the interaction term is $\Delta E^B = g\mu_B m_J B$ (since the good quantum numbers are (H, J, m_J), so the change of energy level is determined by $\Delta E = g\mu_B m_J B$. For the same configuration (same S. L. J), the orginal line will split to $2J+1$ lines with distance $\Delta E = g\mu_B B$. g is different for different configuration.

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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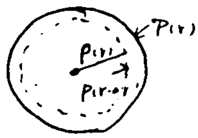
Solution :



(a) Equation for equilibrium

$$\left(P(r-dr)-P(r)\right)\cdot 4\pi r^2 = G\,dm \cdot M(r)/r^2$$

$$dm = \rho\, dv = \rho\, 4\pi r^2 dr$$

(3)

$$\Rightarrow -\frac{dp}{dr} = G\rho\, M(r)/r^2 \quad \checkmark$$

(b) Change scale of the length

$$\underline{\underline{r \to r/y = r'}} \quad \checkmark \quad (y>1)$$

$$\rho = \frac{W\,(\text{mass})}{V\,(\text{volume})} \longrightarrow \rho y^3 = \rho' \quad \checkmark$$

$$M \to M \quad \checkmark$$

$$\Rightarrow$$

$$-\frac{dp'}{dr'} = G\rho' M(r')/r'^2$$

$$-dp' = G\rho y^3 M/\frac{r^2}{y^2} \cdot dr/y$$

$$= G\rho M/r^2 \, dr\, y^4$$

$$\Rightarrow dp' = y^4 dp$$

$$P' = y^4 P \qquad P = nkT \text{ for ideal gas}$$

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$P' = n'kT'$

$n' = y^3 n$

$\Rightarrow T' = yT$

4

$\therefore T' = yT_0$ ✓

$\Delta T = (y-1)T_0$

(c) When the temperature of the star changes, the radiation spectrum shifts.

Now $y>1$, the star contracts, the temperature increases. From the formula

$h\nu_{max} = 2.82\, kT$, we know ν_{max} will increase

$\Rightarrow$ the color of this star changes to higher frequency light like blue.

yes

$L = \sigma T^4 \cdot 4\pi R^2 = 4\pi\sigma R^2 T^4$ yes!

$R' = R/y$

$\frac{L'}{L} = \frac{T'^4 R'^2}{T^4 R^2} = y^4/y^2 = y^2$

3

luminosity increases.

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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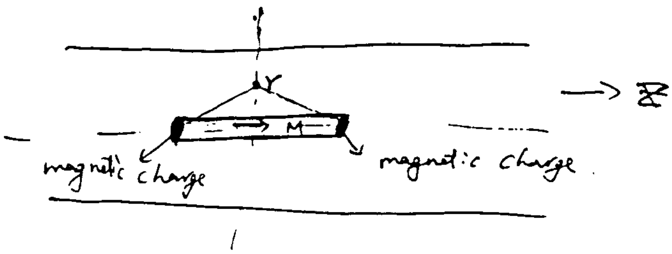
Solution.



(a) ~~$0 < r < a$~~

~~$B = H + 4\pi M$~~

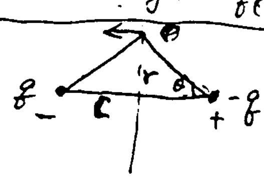
~~$r > a$ ∴ $H = 0$~~

5 points for a reasonable try

$0 < r < a$	$\vec{B} = \vec{H} + 4\pi \vec{M}$
$a < r < b$	$\vec{B} = \vec{H}$

Surface charge density $\sigma = M$

for two point charge q, $-q$



$$H = \frac{2q}{\sqrt{c^2+r^2}} \cdot \cos\theta = \frac{2qc}{c^2+r^2}$$

∴ ~~$H(\theta) = \int H = \int \frac{\sigma \cos\theta}{r^2 + (\frac{l}{2})^2} dS$~~

~~$r^2 = (r - R\cos\theta)^2 + R^2 \sin^2\theta$~~

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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$$\vec{H}(r) = -\hat{e}_z \int \frac{\sigma l}{r'^2 + (\frac{l}{2})^2} ds$$

$0 < r < b$

This is not right because the currents of ~~on~~ the super conductor ($\sigma = M$)

$\cancel{ds = 2\pi R' dR}$ $ds = R dR d\theta$

$0 < R < a$

$0 < \theta < 2\pi$

$$r'^2 = (r - R\cos\theta)^2 + R^2 \sin^2\theta = r^2 + R^2 - 2Rr\cos\theta$$

$$\vec{B}(r) = \begin{cases} \vec{H}(r) & a < r < b \\ \vec{H}(r) + 4\pi M & 0 < r < a \end{cases}$$

(b) $r = b$ $\vec{H}(b) = -H(b)\vec{e}_z$

Since in superconductor $H_t = 0$

$$H_t\big|_{r=b} = \frac{4\pi}{c} j \quad \text{(current per unit length)}$$

$$\vec{j} = \frac{c}{4\pi} H(b)\, \vec{e}_\theta$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution:

(a)

$$\psi_r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \psi_w = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \psi_b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Suppose $R = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

$$R\psi_r = \psi_w \qquad R\psi_w = \psi_b \qquad R\psi_b = 0$$

$\Rightarrow$ $a=0$ $g=0$ $d=1$ $b=0$ $e=0$ $h=1$

$c=0$, $f=0$ $i=0$

$\therefore$ $R = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ ✓

for L:

$$L\psi_b = \psi_w \qquad L\psi_w = \psi_r \qquad L\psi_r = 0$$

$\Rightarrow$

$c'=0$ $i'=0$ $f'=1$

$e'=0$ $h'=0$ $b'=1$

$a'=0$ $d'=0$ $g'=0$

$L = \begin{pmatrix} a' & b' & c' \\ d' & e' & f' \\ g' & h' & i' \end{pmatrix}$

$$L = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

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Solution: they are not physical observable because

$L^+ \neq L, \quad R^+ \neq L.$

3 they don't have real eigenvalues.

(b) $H = \frac{\hbar w}{\sqrt{2}} (R+L)$

$$H = \frac{\hbar w}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$(H-E)|\psi\rangle = 0$

let $E' = \frac{\sqrt{2}}{\hbar w} E$

$\Rightarrow$ $\left(\frac{H}{\hbar w/\sqrt{2}} - E'\right)|\psi\rangle = 0$

3

$\Rightarrow$ $\begin{vmatrix} -E' & 1 & 0 \\ 1 & -E' & 1 \\ 0 & 1 & -E' \end{vmatrix} = 0$

$\Rightarrow$ $E'_1 = 0 \quad E'_2 = \sqrt{2} \quad E'_3 = -\sqrt{2}$

$\Rightarrow$ eigenvalues of H $\Rightarrow$ $H_1 = \hbar w, \; H_2 = 0 \quad H_3 = -\hbar w$

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c. eigenstate of H.

$H_1 = \hbar\omega \Rightarrow \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{2} \\ +1 \end{pmatrix} = |\psi_1\rangle$

$H_2 = 0 \Rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = |\psi_2\rangle$

$H_3 = -\hbar\omega \Rightarrow \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} = |\psi_3\rangle$

$|\psi\rangle\big|_{t=0} = |\psi_w\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(|\psi_3\rangle - |\psi_1\rangle\right)$

$\Rightarrow |\psi(t)\rangle = \frac{1}{\sqrt{2}}|\psi_3\rangle e^{-iE_3t/\hbar} - \frac{1}{\sqrt{2}}|\psi_1\rangle e^{-iE_1t/\hbar}$

find the probability of finding particle in the real section at time t.

$$|\langle\psi_R|\psi(t)\rangle|^2 = \left|\frac{1}{2\sqrt{2}}e^{-iE_3t/\hbar} - \frac{1}{2\sqrt{2}}e^{-iE_1t/\hbar}\right|^2 = \frac{\sin^2\omega t}{2}$$

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Solution:

(a) for temperature T, applied field H. the number of atoms whose magnetic moments are parallel to H is (equilibrium state)

$$N_1 = N \frac{e^{\beta\mu H}}{e^{\beta\mu H} + e^{-\beta\mu H}}$$

for temperature T', applied field $-H$, the number of atoms whose magnetic moments are antiparallel to $-H$ is (equilibrium state)

$$N_1' = N \frac{e^{-\beta'\mu H}}{e^{\beta'\mu H} + e^{-\beta'\mu H}}$$

the energy for getting the atoms to equilibrium.

$$(N_1 - N_1') \cdot 2\mu H$$

Since the system is thermally insulated.

$$(N_1 - N_1') \cdot 2\mu H = Nc \cdot (T - T')$$

$$\Rightarrow$$

$$2\mu H \left(\frac{e^{\beta\mu H}}{e^{\beta\mu H} + e^{-\beta\mu H}} - \frac{e^{-\beta'\mu H}}{e^{\beta'\mu H} + e^{\beta'\mu H}} \right) = c\,(T - T')$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

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Solution:

(a) $|T'-T| << T \Rightarrow \beta' \sim \beta$

$$2\mu H \left(\frac{e^{\beta\mu H} - e^{-\beta\mu H}}{e^{\beta\mu H} + e^{-\beta\mu H}}\right) = -c(T-T') = (\tanh(\beta\mu H)) \cdot 2\mu H$$

$$\Rightarrow \quad T' = T + \frac{\tanh \beta\mu H}{c} \cdot 2\mu H$$

$$T < T'$$

(b) $\frac{\Delta T}{T} > 0.01$

$$\Rightarrow \quad (\tanh \beta\mu H)\, \frac{2\mu H}{cT} > 0.01$$

$$\Rightarrow \quad (\tanh \beta\mu H)\, \beta\mu H\, \frac{2k}{c} > 0.01$$

Choose $c \sim 3k$

$$\Rightarrow \quad (\tanh \beta\mu H)\, \beta\mu H > 0.015$$

~~Suppose $\beta\mu H << 1$ (proved by the result)~~

~~$(\beta\mu H)^2 > 0.015 \Rightarrow \beta\mu H > 0.12$~~

$X = \beta\mu H \quad \Rightarrow \quad f(x) = (\tanh x)\, x \qquad f'(x) = \frac{4}{(e^x+e^{-x})^2} x + \tanh x > 0$

$\Rightarrow$ find $f(x_0) = 0.015$ then for $x > x_0$ $f(x_0) > 0.015$

Note: If you use additional sheets for this problem, number the pages and staple them together.

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$\Rightarrow$ ~~tanh β...~~

$$(\tanh x_0)\, x_0 = 0.015$$

suppose $x \ll 1$ (proved by result)

$$x_0^2 = 0.015$$

$$x_0 = 0.12$$

$\therefore$ $\beta \mu H > 0.12$

initial range of T

$$T < \frac{\mu H}{0.12\, k} = 56\ (K^\circ)$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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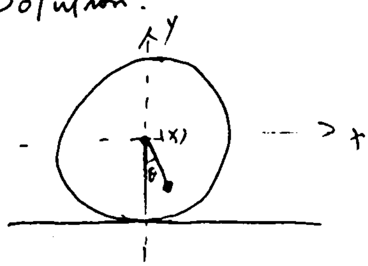
Solution:



(a) Suppose position of center of the wheel is x.

Since the wheel ~~was~~ rolls without slipping, the angular velocity is

$$\omega R = \dot{x}$$

$\Rightarrow$ the kinetic energy of the wheel

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}mR^2 \cdot \frac{\dot{x}^2}{R^2} = m\dot{x}^2$$

the position of the pendulum is

$$(x + l\sin\theta\ ,\ -l\cos\theta)$$

$\Rightarrow$ the kinetic energy of the bob is

$$\frac{1}{2}\mu\,(\dot{x}^2 + l^2\dot{\theta}^2 + 2l\dot{x}\dot{\theta}\cos\theta)$$

potential energy $V = -\mu g l\cos\theta$

$$\Rightarrow \quad L = T - V = m\dot{x}^2 + \frac{1}{2}\mu(\dot{x}^2 + l^2\dot{\theta}^2 + 2l\dot{x}\dot{\theta}\cos\theta) + \cancel{\mu g\cos\theta}\ \mu g l\cos\theta$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution:

(a) Equation of motion.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} \Rightarrow \frac{d}{dt}(2m\dot{x} + \mu\dot{x} + \mu l\dot{\theta}\cos\theta) = 0$$

$$\Rightarrow (2m+\mu)\ddot{x} + \mu l\ddot{\theta}\cos\theta - \mu l\dot{\theta}^2\sin\theta = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{\partial L}{\partial \theta} \Rightarrow \frac{d}{dt}(\mu l^2\dot{\theta} + \mu l\dot{x}\cos\theta) = -\mu l\dot{x}\dot{\theta}\sin\theta - \mu g$$

$$\Rightarrow \mu l^2\ddot{\theta} + \mu l\ddot{x}\cos\theta - \mu l\dot{x}\dot{\theta}\sin\theta = -\mu l\dot{x}\dot{\theta}\sin\theta$$

$$\Rightarrow \mu l^2\ddot{\theta} + \mu l\ddot{x}\cos\theta = -\mu g l\sin\theta \qquad \langle 2 \rangle$$

(b) from (1) $\Rightarrow$ θ is small

$$2m\dot{x} + \mu\dot{x} + \mu l\dot{\theta}\cos\theta = \text{const}.$$

$$\Rightarrow 2m\dot{x} + \mu\dot{x} + \mu l\dot{\theta} = \text{const} = A \qquad (3)$$

$$\mu l^2\ddot{\theta} + \mu l\ddot{x} = -\mu g l\theta \qquad (4)$$

from (3) $\Rightarrow$ $(2m+\mu)\ddot{x} = -\mu l\ddot{\theta}$

$$\Rightarrow \frac{2ml}{2m+\mu}\ddot{\theta} = -g\theta$$

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Solution: (b)

$$\theta = \theta_0' \cos(\omega t + \varphi_0)$$

θ_0', φ_0 determined by ~~initial~~ initial condition

$$\omega = \sqrt{\frac{g}{l}\left(\frac{2m+M}{2m}\right)}$$

from 3.

$$\Rightarrow \quad (2m+M)\dot{x} + Ml\dot{\theta} = A$$

$$\Rightarrow \quad \dot{x} = \frac{1}{2m+M}(A - Ml\dot{\theta})$$

$$\Rightarrow \quad x - x_0 = \frac{1}{2m+M}\left[At - Ml(\theta - \theta_0)\right]$$

(c) $m \gg M \qquad \omega \approx \sqrt{g/l}$

~~That~~ corresponds to the condition that the wheel is fixed.

$m \ll M \qquad \omega \sim 0$

That corresponds to the ~~condition~~ the the bob is too heave that the wheel can't ~~make~~ it to oscillate.

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution.

(d)

Since

$$(2m+\mu)\dot{x} + \mu l\dot{\theta} = \text{const}. \quad x - x_0 = \frac{1}{2m+\mu}(At-$$

⇒ that is, except a steady move of x, let $x' = x$

$$x' - x_0' = -\frac{\mu}{2m+\mu} l(\theta - \theta_0)$$

the wheel and the bob oscillate with each

and the phases of them are just the oppos

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution:

(a)

$$zJ'' + J' + zJ(z) = 0$$

$$J(0) = 1$$

$$J(z) = 1 + az + bz^2 + cz^3 + dz^4 + \cdots$$

put into the equation

$$12dz^3 + 6cz^2 + 2bz + a + 2bz + 3cz^2 + 4dz^3 + z + az^2 + bz^3 + \cdots = 0$$

$\Rightarrow$ $a = 0$ ✓ Compare coefficience of the same power of z

$b = -\frac{1}{4}$

$c = 0$

$d = \frac{1}{64}$

$$\Rightarrow \quad J(z) = 1 - \frac{1}{4}z^2 + \frac{1}{64}z^4 + \cdots$$

(b) $$F(t) = \int_0^{\frac{\pi}{2}} \exp\{t\cos\theta\}\theta^2 J(\theta)\,d\theta.$$

when $t \to \infty$ the main contribution of the integral comes from θ.

$$F(t) = \int_0^{\frac{\pi}{2}} \exp\{t(1-\frac{\theta^2}{2})\}\,\theta^2 (1-\frac{1}{4}\theta^2)\,d\theta$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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(b)

$$F(t) = \exp t \int_0^{\frac{\pi}{2}} \exp\left(-\frac{t\theta^2}{2}\right) \theta^2 (1 - \tfrac{1}{4}\theta^2) d\theta$$

$$= \exp t \int_0^{\frac{\pi}{2}} \exp\left(-\frac{t\theta^2}{2}\right) \theta^2 \, d\theta \, (1 - \tfrac{1}{4}\theta^2) d\theta$$

first term (neglect $\frac{1}{4}\theta^2$) $\cancel{= e^t \int_0^{\frac{\pi}{2}} \exp(-\theta'^2)\, \theta'^2 \, d\theta'}$

$$F(t)_1 = \frac{e^t}{(\sqrt{t/2})^3} \int_0^{\frac{\pi}{2}\sqrt{\frac{t}{2}}} \exp(-\theta'^2)\, \theta'^2 \, d\theta'$$

$$\overset{t\to\infty}{=} \frac{e^t}{(\sqrt{t/2})^3} \int_0^{\infty} \exp(-x^2)\, x^2 \, dx$$

$$= \frac{e^t}{t^{3/2}} 2^{3/2} \left(\frac{1}{2} \Gamma\left(\frac{3}{2}\right) \right)$$

$$= \frac{\sqrt{\pi}}{4} \cdot 2^{3/2} \, e^t / t^{3/2} = \sqrt{\frac{\pi}{2}} \, \frac{e^t}{t^{3/2}}$$

Second term

$$F(t)_2 = - e^t \int_0^{\frac{\pi}{2}} \exp\left(-\frac{t\theta^2}{2}\right) \theta^2 \, \frac{1}{4}\theta^2 \, d\theta$$

$$= - \frac{e^t}{4 (\sqrt{\frac{t}{2}})^5} \int_0^{\frac{\pi}{2}\sqrt{\frac{t}{2}}} \exp(-x^2)\, x^4 \, dx$$

$$= - \frac{e^t}{4 t^{5/2}} 2^{5/2} \cdot \int_0^{\infty} \exp(-x^2)\, x^4 \, dx$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution :

$$F(t)_2 = -\frac{e^t}{8t^{5/2}} 2^{5/2} \Gamma\left(\frac{5}{2}\right) = -\frac{e^t}{t^{5/2}} \frac{3\sqrt{2\pi}}{8}$$

$\therefore$ first two terms are

$$F(t) = \sqrt{\frac{\pi}{2}} \frac{e^t}{t^{3/2}} - \frac{e^t}{t^{5/2}} \frac{3\sqrt{2\pi}}{8}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 10

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Solution:

(a)

$$\mathcal{L} = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 - U(r)$$

$$\Rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot\theta} = 0 \Rightarrow mr^2\dot{\theta} = \text{const} = M$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot r} = \frac{\partial L}{\partial r} \Rightarrow m\ddot{r} = -\frac{\partial U(r)}{\partial r} + mr\dot{\theta}^2$$

$$= \cancel{F(r)} - \frac{\partial U(r)}{\partial r} + \frac{M^2}{mr^3}$$

$$= -\frac{\partial}{\partial r}\left(U(r) + \frac{M^2}{2mr^2}\right)$$

$$\Rightarrow U_{eff} = U(r) + \frac{M^2}{2mr^2} \qquad \frac{\partial U(r)}{\partial r} = -F(r)$$

$$U(r) = -\frac{1}{\beta}\frac{k}{r^\beta}$$

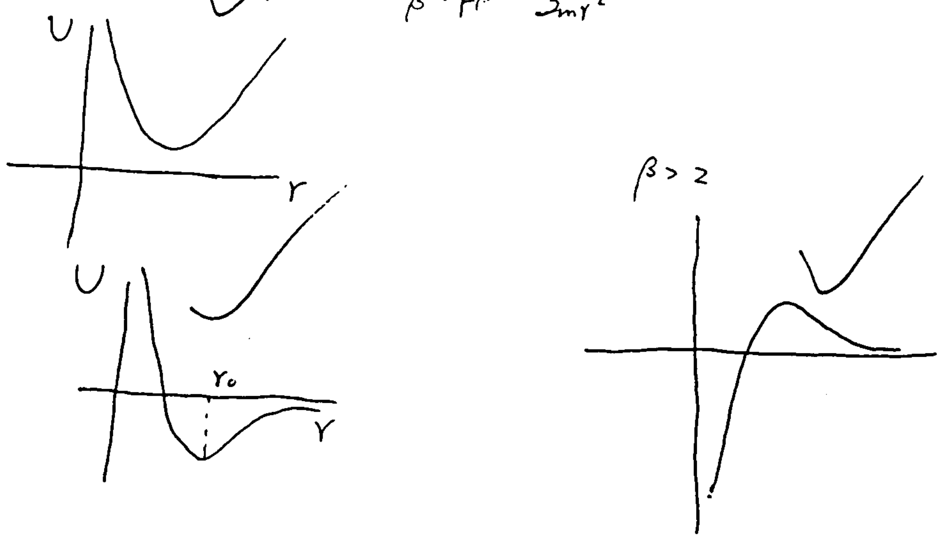
$$\Rightarrow U_{eff} = -\frac{1}{\beta}\frac{k}{r^\beta} + \frac{M^2}{2mr^2}$$

(b)

$\beta<0$

$2>\beta>0$

$\beta>2$



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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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for $\beta < 0$, all orbits bounded.

for $2 > \beta > 0$, and $\beta<0$ a stable circular orbit exist.

~~and $\beta<0$~~

(c) for $2 > \beta > 0$ and $\beta < 0$ r_0 corresponds to the minimum value of

$$\frac{d\, V_{eff}(r)}{dr} = 0$$

~~for $\beta<0$~~

$$\Rightarrow \quad \frac{k}{r_0^{\beta+1}} - \frac{M^2}{m r_0^3} = 0$$

$$\Rightarrow \quad r_0^{2-\beta} = \frac{M^2}{mk}$$

$$\Rightarrow \quad r_0 = \left(\frac{M^2}{mk}\right)^{\frac{1}{2-\beta}}$$

(d) $r = r_0 + \eta$ put into equation of r.

$$m\ddot{r} = \frac{-k}{r^{\beta+1}} + \frac{M^2}{mr^3}$$

expand at $r = r_0 + \eta$

$$\Rightarrow \quad m\ddot{\eta} = \frac{-k}{r_0^{\beta+1}}\left[1 - (\beta+1)\frac{\eta}{r_0}\right] + \frac{M^2}{mr_0^3}\left[1 - 3\frac{\eta}{r_0}\right)$$

$$= \left[-\frac{k(\beta+1)}{r_0^{\beta+1}}\frac{1}{r_0} + \frac{3M^2}{mr_0^4}\right]\eta$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution.

$$\Rightarrow \quad m\ddot{\eta} = -\left[-\frac{k(\beta+1)}{r_0^{\beta+2}} + \frac{3M^2}{m r_0^4}\right]\eta = -\left[\frac{M^2}{m r_0^4}(2-\beta)\right]\eta$$

where $r_0 = \left(\frac{M^2}{mk}\right)^{\frac{1}{2-\beta}}$

When the orbit is closed, the frequency of η ~~should~~ should satisfied.

$$\frac{\omega_\eta}{\omega_\theta} = \frac{m}{n} \quad \text{(where } m, n \text{ are interger)}$$

$$\cancel{\omega_\eta = \left(\frac{-k(\beta+1)}{r_0^{\beta+2}} + \frac{3M^2}{m r_0^4}\right)^{\frac{1}{2}}} \qquad \omega_\eta = \frac{M}{m r_0^2}(2-\beta)^{\frac{1}{2}}.$$

$$\omega_\theta = \dot{\theta} = \frac{M}{m r_0^2}$$

$$\Rightarrow \quad \beta = 2$$

or $2-\beta = n^2 \Rightarrow n = 1, 2, 3 \ldots\ldots$ (really want $\sqrt{2-\beta} = \frac{m}{n}$ any rational #)

(almost) perfect!

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = $9\frac{1}{2}$

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the Problem No. 12-1 and your Identification No. . 59

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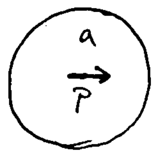

Solution: outside $\nabla^2\phi_2 = 0$

inside $\phi = \phi_1 + \phi_0$ $\vec{P} = P\vec{e_z}$

ϕ_0 is the ~~field~~ potential of dipole

ϕ_1 satisfied $\nabla^2\phi_1 = 0$

$$\Rightarrow \quad \phi_0 = -\frac{p}{\varepsilon}\cdot\nabla\frac{1}{r} = \frac{\vec{p}\cdot\vec{r}}{\varepsilon r^3} = \frac{p\cos\theta}{\varepsilon r^2}$$

$$\phi_1 = \sum\left(a_n r^n + \frac{b_n}{r^{n+1}}\right) P_n(\cos\theta)$$

$$\phi_2 = \sum\left(c_n r^n + \frac{d_n}{r^{n+1}}\right) P_n(\cos\theta)$$

$\Rightarrow$ for ϕ_1, $r=0$ ϕ_1 limited

for ϕ_2, $r=\infty$ $\phi_2 = 0$

for $r=a$ $\phi_2 = \phi_1 + \phi_0$

$r=a$ $-\frac{\partial\phi_2}{\partial r} = -\frac{\partial(\phi_1+\phi_0)}{\partial r}\varepsilon$

$$\Rightarrow \quad \phi_2 = \frac{d}{r^2}\cos\theta$$

$$\phi = \phi_0 + \phi_1 = a_1 r\cos\theta + \frac{P\cos\theta}{\varepsilon r^2}$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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$$\begin{cases} \frac{d}{a^2} = a_1 a + \frac{P}{\varepsilon a^2} \\ \frac{2d}{a^3} = -\varepsilon\left(a_1 - \frac{2P}{\varepsilon a^3}\right) \end{cases}$$

$$\Rightarrow \quad a_1 = \frac{P}{a^3}\left(1-\frac{1}{\varepsilon}\right)\left(\frac{2}{2\varepsilon+1}\right)$$

$$\Rightarrow \quad d = \frac{P}{\varepsilon}\,\frac{4\varepsilon-1}{2\varepsilon+1}$$

$\therefore$ Electric field

Outside. $\vec{E} = -\nabla\phi_2 = -\nabla \frac{P\cos\theta}{r^2}\,\frac{4\varepsilon-1}{(2\varepsilon+1)}\,\frac{1}{\varepsilon}$

$$= \frac{4\varepsilon-1}{(2\varepsilon+1)}\,\frac{1}{\varepsilon}\left[-\nabla\left(\vec{P}\cdot\frac{\vec{r}}{r^3}\right)\right]$$

$$= -\frac{4\varepsilon-1}{(2\varepsilon+1)}\,\frac{1}{\varepsilon}\,\frac{r^2\vec{P} - 3\vec{r}(\vec{r}\cdot\vec{P})}{r^5}$$

inside. $\vec{E} = -\nabla(\phi_1+\phi_0) = -\nabla\left(a_1 r\cos\theta + \frac{P\cos\theta}{\varepsilon r^2}\right)$

$$= -a_1\vec{e}_z + \frac{1}{\varepsilon}\left(-\nabla\frac{\vec{P}\cdot\vec{r}}{r^3}\right)$$

$$= -\frac{\vec{P}}{a^3}\left(1-\frac{1}{\varepsilon}\right)\left(\frac{2}{2\varepsilon+1}\right) - \frac{1}{\varepsilon}\,\frac{r^2\vec{P} - 3\vec{r}(\vec{r}\cdot\vec{P})}{r^5}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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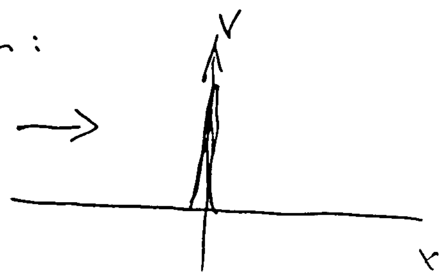
Solution:

(a)



$x<0$ $\psi_1 = e^{ikx} + A e^{-ikx}$ $k^2 = \frac{2mE}{\hbar^2}$. (from schrodinger equation

$x>0$ $\psi_2 = B e^{ikx}$ $\frac{d^2\psi}{dx^2} + \left(\frac{2m}{\hbar^2}(E-V)\right)\psi = 0$

$\Rightarrow$ $x=0$ $\psi_1 = \psi_2$

$\Rightarrow$ $1 + A = B$ (1)

$x=0$

$$\frac{d\psi_2}{dx} - \frac{d}{dx}\psi_1 + \int_{-\delta}^{\delta} \frac{2m}{\hbar^2}\left(-\frac{\hbar^2}{2m}\beta \cdot \delta(x)\right)\psi = 0$$

$$\Rightarrow \frac{d\psi_2}{dx} - \frac{d\psi_1}{dx} = \beta\psi(0)$$

$$ikB - (ik - ikA) = \beta B \quad (2)$$

$$\Rightarrow \quad A = \frac{\beta}{2ik-\beta} \qquad B = 1 + A = \frac{2ik}{2ik-\beta}$$

$\Rightarrow$ Reflection coefficient $R = |A|^2 = \frac{\beta^2}{\beta^2+4k^2}$

Transmision .. $T = |B|^2 = \frac{4k^2}{4k^2+\beta}$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score =

Please insert on page

the Problem No. 13-2 and your Identification No. . 59

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(b) for negative β

bound state $E<0$

$x<0 \quad \psi_1 = A' e^{k'x}$ $\qquad k'^2 = \frac{2m|E|}{\hbar^2}$

$x>0 \quad \psi_2 = B' e^{-k'x}$

at $x=0 \qquad \psi_1 = \psi_2$

$A' = B' \quad (1)$

$$\frac{d\psi_2}{dx} - \frac{d\psi_1}{dx} = \beta\psi(0)$$

$\Rightarrow \quad -k'B' - kA' = \beta B' \quad (2)$

$\Rightarrow \quad 2k'B' = -\beta B'$

$k' = \frac{-\beta}{2} \Rightarrow k'^2 = \frac{\beta^2}{4} = \frac{2m|E|}{\hbar^2}$.

$\Rightarrow |E| = \frac{\hbar^2\beta^2}{8m}$.

$\Rightarrow$ bound state $\begin{cases} \psi_1 = A' e^{k'x} & x<0 \\ \psi_2 = B' e^{-k'x} & x>0 \end{cases}$ where $k = -\frac{\beta}{2}$.

Scattering state $E>0$ the same as in (a).

$\begin{cases} \psi = e^{ik_0x} + A e^{-ik_0x} & x<0) \\ \psi_2 = B e^{ik_0x} & (x>0) \end{cases}$ $\qquad k_0^2 = \frac{2mE}{\hbar^2}$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score =

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the Problem No. 13-3 and your Identification No. . 59

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Solution :

$$\langle \psi_s | \psi_b \rangle \sim \int_0^\infty B^* e^{-ik^* x} e^{-k'x} dx + \int_{-\infty}^0 e^{+k'x} (A^* e^{+ik^* x} + e^{-ik^* x}$$

$$= B^* \frac{1}{k' + k^* i} + A^* \frac{1}{k' + k^* i} \cancel{\frac{1}{k \mp k'}} + \frac{1}{k' - ik^*} \qquad \langle 1 \rangle$$

put

$$A^* = \frac{\beta}{-2ik - \beta} \qquad B^* = \frac{-2ik}{-2ik - \beta}$$

$$k' = -\frac{\beta}{2} \quad \text{into} \quad \langle 1 \rangle \Rightarrow \langle \psi_s | \psi_b \rangle = 0$$

↳ orthogonal

(c) for negative β $\quad A = \frac{\beta}{2ik - \beta}$, $B = \frac{2ik}{2ik - \beta}$ there is a pole for $k = \frac{-\beta}{2} i$

that correspond to if $k = -\frac{\beta}{2} i$, $\quad B = \infty$, $A = \infty$,

that is a bound state.

~~Since for bound state~~ .

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 10

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the Problem No. 14-1 and your Identification No. . 59

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Solution :

(1) for fermions

$$(2S+1)\int_0^{\varepsilon_F} (g(\varepsilon))\, d\varepsilon = N \qquad S=\tfrac{1}{2}$$

$$(2S+1)\,\frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\frac{2}{3}\,\varepsilon_F^{\frac{3}{2}} = N \qquad \text{let } \frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} = K$$

$$\varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{\frac{2}{3}} = \left(\frac{N}{K}\,\frac{3}{4}\right)^{2/3}$$

$$E = (2S+1)\int_0^{\varepsilon_F} \varepsilon\, g(\varepsilon)\, d\varepsilon = 2\cdot\frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\frac{2}{5}\,\varepsilon_F^{\frac{5}{2}}$$

$$\underset{(2)}{=} \frac{3}{5}N\left(\frac{N}{K}\,\frac{3}{4}\right)^{2/3} = \frac{N^{\frac{5}{3}}}{K^{\frac{2}{3}}}\cdot 0.495$$

The system is insulated , so the total energy for bosons also (2) is E

for Bose - condense $\mu = 0$

$$E_b' = \int_0^\infty \varepsilon\, g(\varepsilon)\, d\varepsilon\, \frac{1}{e^{\beta\varepsilon}-1} \qquad N = \int_0^\infty g(\varepsilon)\, d\varepsilon\, \frac{1}{e^{\beta\varepsilon}-1}$$

$$= K'\beta^{-5/2}\cdot 1.78 \qquad = K'\int_0^\infty \frac{\varepsilon^{1/2}}{e^{\beta\varepsilon}-1}\, d\varepsilon$$

$$= K'\left(\frac{N}{K'}\,\frac{1}{2.32}\right)^{5/3}\cdot 1.78 \qquad = K'\beta^{-3/2}\cdot 2.32$$

$$= \frac{N^{5/3}}{K'^{2/3}}\cdot 0.4387 \qquad \beta = \left(\frac{N}{K'}\,\frac{1}{2.32}\right)^{-2/3}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score =

Please insert on page

the Problem No. 14-2 and your Identification No. . 59

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~~Since $E_b' < E$~~

~~$\Rightarrow$ no bo~~

let $E_b' = E$

$\Rightarrow \quad K' < K$

$\Rightarrow \quad V' < V$

That is the critical volumn ~~for~~ ~~bose~~ - condense is smaller than the volume now. So no bose-conde

✔

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 10

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the Problem No. 15-1 and your Identification No. 59

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Solution:

$$H = \frac{P_1^2}{2m} + \frac{P_2^2}{2m} + \cdots \frac{P_n^2}{2m} + \sum_{i<j} V(\vec{r}_i - \vec{r}_j)$$

$$= -\frac{\hbar^2}{2m}(\nabla_1^2 + \cdots \nabla_n^2) + \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|)$$

$$\frac{\partial E_n}{\partial \hbar} = \langle \psi_n | \frac{\partial H}{\partial \hbar} | \psi_n \rangle$$ (Feynman - Helmann theorem)

$$= \langle \psi_n | -\frac{\hbar}{m}(\nabla_1^2 + \cdots \nabla_n^2) | \psi_n \rangle$$

$$= \overline{E}_{kin} \cdot \frac{2}{\hbar}$$

Change ~~tot~~ length scale: $r \to r/\hbar = R$ $\quad \nabla_1' = \frac{d}{d\vec{R}_1}$

then

$$H' = -\frac{1}{2m}(\nabla_1'^2 + \cdots \nabla_n'^2) + \sum_{i<j} V[|\vec{R}_i - \vec{R}_j|\hbar]$$

$$= -\frac{1}{2m}(\nabla_1'^2 + \cdots \nabla_n'^2) + \sum_{i<j} \frac{1}{\hbar} V(|\vec{R}_i - \vec{R}_j|)$$

$$\frac{\partial E_n'}{\partial \hbar} = \langle \psi_n' | \frac{\partial H'}{\partial \hbar} | \psi_n' \rangle = -\frac{1}{\hbar^2} \langle \psi_n' | \sum_{i<j} V(|\vec{R}_i - \vec{R}_j|) \ldots$$

$$= -\frac{1}{\hbar} \overline{E}_{pot}'$$

We know that for physical system, its energy levels do not ... on the length scale.

$$\Rightarrow \quad \frac{\partial E_n}{\partial \hbar} = \frac{\partial E_n'}{\partial \hbar} \qquad \overline{E}_{pot}' = \overline{E}_{pot}.$$

$$\Rightarrow \quad 2\overline{E}_{kin} + \overline{E}_{pot} = 0$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 10

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the Problem No. 16-1 and your Identification No. 59

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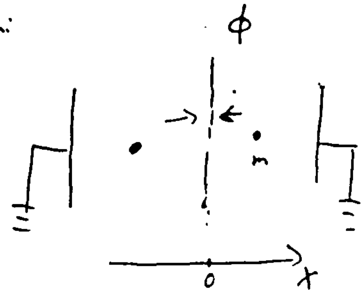
Solution:



$$E = \frac{\phi}{d}$$

$$m\ddot{x} = eE$$

$$= eE \left(1 - \frac{\vec{x}}{x}\right)$$

Now calculate the adiabatic invariant

$$I = \oint p\, dq = 4\int_{A_0}^{0} p\, dx = 4\int_0^{A_0} m\dot{x}\, dx$$

$$= 4\int_0^{A_0} m\dot{x}\, dx$$

x from $A_0 \to 0$

$$m\ddot{x} = -eE$$

$$m\dot{x} = -eEt \qquad t = \frac{-m\dot{x}}{eE}$$

$$mx = mx_0 - \frac{1}{2}eEt^2$$

$$\dot{x}^2 = \frac{2eE}{m}(A_0 - x) \qquad \dot{x} = \sqrt{\frac{2eE}{m}}\sqrt{A_0 - x}$$

$$I = 4\sqrt{\frac{2eE}{m}}\, m \int_0^{A_0} \sqrt{A_0 - x}\, dx = \frac{8}{3}\sqrt{2eEm}\, A_0^{3/2}$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score =

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the Problem No. 16-2 and your Identification No. 19

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Solution: $\cdot$ I is invariant.

$$E^{\frac{1}{2}} A_0^{3/2} = E'^{\frac{1}{2}} A_0'^{3/2}$$

$$E A_0^3 = E' A_0'^3$$

$$\frac{E'}{E} = \frac{\Phi'}{\Phi} = 2 = \left(\frac{A_0^3}{A_0'^3}\right)$$

$$\underline{\underline{A' = A_0\, 2^{-1/3}}}$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

Score = ~~8~~ 10

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Please insert on page the Problem No. 17-1 and your Identification No. 59

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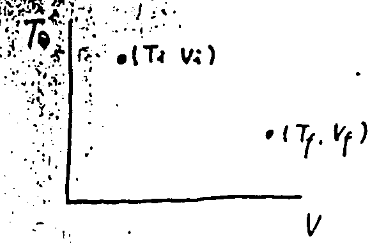
Solution



(a) $P = A \cdot N^2 V^{-2} T^3 \qquad C_v = B N^4 V^{-3} T$

~~$\Delta W = 0 \Rightarrow \Delta E = \Delta Q$~~

$$dE = T\,ds - P\,dV$$
$$= C_v\,dT + \left(T\left(\frac{\partial P}{\partial T}\right)_v - P\right)dV$$
$$= B N^4 V^{-3} T\,dT + (2 A N^2 V^{-2} T^3)\,dV$$

$$E - E_0 = \int_{T_0 V_0}^{T_f V_f} dE = \left[\int_{T_0\ V_0}^{T_f\ V_0} + \int_{T_f\ V_0}^{T_f\ V_f}\right] dE$$

$$= \frac{B}{2} N^4 V_0^{-3}\left(T_f^2 - T_i^2\right) + 2 A N^2 T_f^3 \left(-\frac{1}{V_f} + \frac{1}{V_0}\right)$$

$$\Rightarrow \Delta E = \frac{B}{2} N^4 V_0^{-3}(T_f^2 - T_i^2) + 2 A N^2 T_f^2 \left(-\frac{1}{V_f} - \frac{1}{V_0}\right)$$

$$\Delta E = \Delta Q - \Delta W = 0$$

(b) Since dE must be a state function

$$\frac{\partial}{\partial V}\frac{\partial E}{\partial T} = \frac{\partial}{\partial T}\left(\frac{\partial E}{\partial V}\right) \Rightarrow -3 B N^4 V^{-4} T = 6 A N^2 V^{-2} T^2$$

$$\Rightarrow T = \frac{-B N^2}{2 A V^2}$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 8

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the Problem No. 17-1 and your Identification No. . 59

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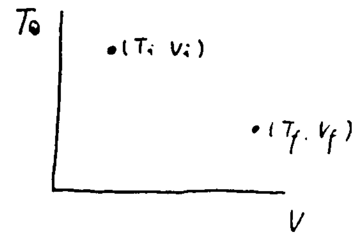
Solution:



(a) $P = A \cdot N^2 V^{-2} T^3 \qquad C_v = B N^4 V^{-3} T$

$\cancel{\Delta W = 0 \Rightarrow \Delta E = \Delta Q}$

$$dE = T\,ds - p\,dV$$

$$= C_v\,dT + \left(T\left(\frac{\partial P}{\partial T}\right)_v - P\right) dV$$

$$= B N^4 V^{-3} T\,dT + (2 A N^2 V^{-2} T^3)\,dV$$

$$E - E_0 = \int_{T_i V_0}^{T_f V_f} dE = \left[\int_{T_0 V_0}^{T_f V_0} + \int_{T_f V_0}^{T_f V_f}\right] dE$$

$$= \frac{B}{2} N^4 V_0^{-3} (T_f^2 - T_i^2) + 2 A N^2 T_f^3 \left(-\frac{1}{V_f} + \frac{1}{V_0}\right)$$

$\Rightarrow$

$$\Delta E = \frac{B}{2} N^4 V_0^{-3} (T_f^2 - T_i^2) + 2 A N^2 T_f^2 \left(-\frac{1}{V_f} - \frac{1}{V_0}\right)$$

$$\Delta E = \Delta Q - \Delta W = 0$$

(b) Since dE must be a state function

$$\frac{\partial}{\partial V}\frac{\partial E}{\partial T} = \frac{\partial}{\partial T}\left(\frac{\partial E}{\partial V}\right) \Rightarrow -3 B N^4 V^{-4} T = 6 A N^2 V^{-2} T^2$$

$$\Rightarrow T = \frac{-B N^2}{2 A V^2}$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 3

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the Problem No. 18 − 1 and your Identification No. 59

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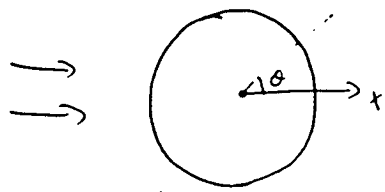
Solution:



$$\nabla^2 \vec{E}_s - \frac{1}{c^2}\frac{\partial^2 \vec{E}_s}{\partial t^2} = 0$$

$$\vec{E}_s = E(r,\theta)\,\hat{z}$$

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

$$\cancel{E_s = \frac{f(\theta)}{r} e^{ikr}}$$

$$\Rightarrow \nabla^2 E + \frac{w^2}{c^2} E = 0$$

let $E = E(r) f(\theta)$

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{dE(r)}{dr}\right) f(\theta) + \frac{E(r)}{r^2}\frac{d^2 f(\theta)}{d\theta^2} + \frac{\omega^2}{c^2} E(r) E(\theta) = 0$$

$$\Rightarrow \frac{d^2 f(\theta)}{d\theta^2} = -n^2 f(\theta)$$

$$\therefore \frac{1}{r}\frac{d}{dr}\left(r\frac{dE(r)}{dr}\right) + \frac{\omega^2}{c^2} E(r) - n^2 \frac{E(r)}{r^2} = 0$$

$$\frac{d^2}{dr^2}E(r) + \frac{1}{r}\frac{dE(r)}{dr} + \left(\frac{\omega^2}{c^2} - \frac{n^2}{r^2}\right)E(r) = 0$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score =

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the Problem No. 18-2 and your Identification No. . 59

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$f(\theta) = A_n \cos n\theta + B_n \sin n\theta$ (for periodical boundary reason

$n = 0, \pm 1, \pm 2, \pm 3 \ldots$

let $r' = \frac{\omega}{c} r = kr$

$\Rightarrow \quad \frac{d^2 E(r')}{dr'^2} + \frac{1}{r'}\frac{dE(r')}{dr'} + (1 - \frac{n^2}{r'^2})E(r') = 0$

$\Rightarrow$ Bessel function.

$E(r') = C_n J_n(r') + D_n N_n(r)$

no outgoing wave for scatter wave

$r' \to \infty \quad E(r') = 0 \quad C_n = 0$

$\Rightarrow \quad E(r') = D_n N_n(r')$

$\Rightarrow \quad \vec{E}_s = \sum_n \left(D_n N_n(r') (A_n \cos n\theta + B_n \sin n\theta) \right) e^{-i\omega t} \,) \hat{e}_z$

$r = a$ boundary condition

$\vec{n} \times (\vec{E}_{in} + \vec{E}_s)\Big|_{r=a} = 0$

$\Rightarrow \quad E_0 e^{ika\cos\theta} = -\sum_n N_n(\frac{\omega}{c}a)\,(A_n' \cos n\theta + B_n' \sin n\theta))$

$\Rightarrow \quad A_n' = \quad B_n' = 0 \quad (\theta \to -\theta)$

$\Rightarrow \quad \vec{E}_s = \sum_{n=0}^{\infty} A_n' N_n(kr) \cos n\theta \, e^{-i\omega t} \, \hat{e}_z$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score =

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the Problem No. 18 - 3 and your Identification No. . 59

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(b) $\sigma = \frac{P_s}{I_{in}} = \frac{\int \bar{S} \cdot r\,dr\,d\theta\, L}{c \cdot u}$ → length

$$u = \overline{\frac{E^2}{8\pi} + \frac{H^2}{8\pi}} = \frac{E_0^2}{8\pi}$$

$|\vec{H}| = |\vec{E}|$

$$\bar{S} = \frac{c}{4\pi} \vec{E} \times \vec{H} = \frac{c}{4\pi} E_s^2 = \frac{c}{8\pi} |E_s|^2$$

(1)

$$E_s = E(r) f(\theta)\, e^{i\omega t}$$

$$\cancel{\sigma = \frac{\int E^2(r) f^2(\theta)\, r\,dr\,d\theta}{E_0^2}}$$

$0 < \theta < 2\pi$

$$\sigma = \frac{\int |E_s|^2\, r\,dr\,d\theta}{E_0^2}$$

(c) if $ka \ll 1$

E_0: only need keep two terms $n=0,\ n=1$

(1)

$$E_0(1 + ika\cos\theta) = A_0' N_0(ka) + A_1' N_1(ka)\cos\theta$$

$$A_0' = \frac{E_0}{N_0(ka)} \qquad A_1' = \frac{ika\,E_0}{N_1(ka)}$$

$$\sigma = \int \left| \frac{N_0(kr)}{N_0(ka)} + \frac{ika}{N_1(ka)} N_1(kr)\cos\theta \right| r\,dr\,d\theta$$

$=$

$ka \ll 1$ ⇒ corresponds to dipole radiation.

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 9

Please insert on page

the Problem No. 19-1 and your Identification No. . 59

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Solution:

$$\frac{\partial \psi}{\partial t} - \frac{\partial^2 \psi}{\partial x^2} + 8\psi = \delta(x-2t)$$

use $\int_{-\infty}^{+\infty} e^{-ikx} dx$ in both sides of the equation. $\int_{-\infty}^{+\infty} e^{-ikx} \cdot \psi dx =$

$$\Rightarrow \quad \frac{\partial \psi_k}{\partial t} + k^2 \psi_k + 8\psi_k = e^{-ik\cdot 2t}$$

$\psi_k = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ik}$

$$\Rightarrow \quad \frac{\partial \psi_k}{\partial t} + (k^2+8)\psi_k = e^{-ik\cdot 2t}$$

$\int_{-\infty}^{+\infty} e^{-ikx} e^{ikx'} dk$

$= 2\pi \delta$

$$\psi_k = c\, e^{-(k^2+8)t} + \frac{e^{-2ikt}}{k^2+8-2ki}$$

$\psi(\pm\infty) = 0$

Choose $C=0$

(particular) solution:

$$\psi = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ikx} \psi_k \, dk$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{e^{-2ikt}}{k^2+8-2ki} e^{ikx} \, dk$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{e^{ik(x-2t)}}{k^2+8-2ki} \, dk$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{e^{ik(x-2t)}}{(k-4i)(k+2i)} \, dk$$

Note: If you use additional sheets for this problem, number the pages and staple them together.

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score =

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the Problem No. 19-2 and your Identification No. 59

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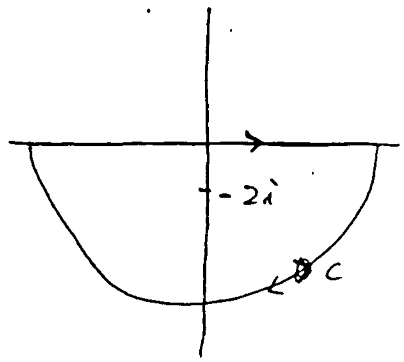
if $x < 2t$ choose contour

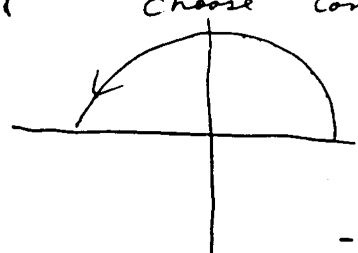


k-space.

$$\psi(x,t) = \frac{1}{2\pi} \cdot 2\pi i \; \frac{e^{2(x-2t)}}{-6i}$$

$$= -\frac{1}{6} e^{2(x-2t)}$$

if $x > 2t$ choose contour



$$\psi(x,t) = \frac{1}{2\pi} \, 2\pi i \; \frac{e^{-4(x-2t)}}{6i} = \frac{1}{6} e^{-4(x-2t)}$$

$$\Rightarrow \quad \psi = \begin{cases} \frac{1}{6} e^{-4(x-2t)} & x > 2t \\ -\frac{1}{6} e^{2(x-2t)} & x < 2t \end{cases}$$

Didn't check on direction of circling.

Note: If you use additional sheets for this problem, number the pages and staple them together.