DEPARTMENT OF PHYSICS
University of California, San Diego
La Jolla, California 92093-0319

WRITTEN DEPARTMENTAL EXAMINATION - SPRING, 1989

PART I

Each problem is worth 10 points.

Problem 1

An electric dipole of strength p is oriented perpendicular to and at a distance d from an infinite conducting plane. Calculate the force exerted on the plane by the dipole.

Problem 2

Give approximate numerical values for the following (be sure to give units):

a. Escape velocity from the earth
b. Ground state energy of muonium [μ_+,μ_- atom]
c. $\sum \frac{(-1)^{n-1}}{n}$
d. Spin of the deuteron
e. Lifetime of the neutron
f. Specific heat of Cu at room temperature (assumed greater than Debye temperature)
g. The orbital period of a planet (Saturn) which is 3 times as far from the sun as the earth is.

Problem 3

A hydrogen atom is placed in a uniform electric field $E(t)$ in the z-direction. $E(t)$ is 0 for $t<0$ and $E(t)=E_0 e^{-t/\tau}$ for $t>0$. The atom is initially in the ground state. Find the probability for the atom to have made a transition to the 2S state as $t \rightarrow \infty$. Find the probability for the atom to have made a transition to the 2P state as $t \rightarrow \infty$. (Assume E_0 is small. How small should it be for the validity of your calculation?)

Problem 4

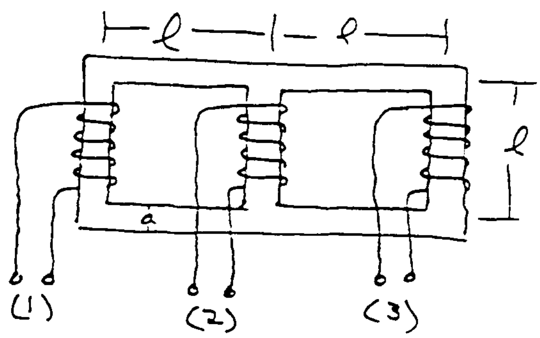
A magnetic circuit consists of a set of three straight parallel segments of length l and cross sectional area a^2, spaced from one another by l; these are then connected at their ends by an additional four similar segments, as shown. Their material has a relative permeability $\mu_r \gg 1$.

Each of the three parallel legs is closely wound with a coil of n turns. For purposes of this problem, assume that all of the flux from each coil is confined entirely to the magnetic core. From left to right in the diagram, we designate them as coils 1, 2, and 3. Neglect coil resistance.

a. If a current I_1 is passed through coil 1, with $I_2 = I_3 = 0$, calculate the magnetic field induced in leg 3.

b. If a voltage $\mathcal{E}$ is applied to coil 1 while coils 2 and 3 are open, what is the voltage across coil 2?

c. For the conditions of part a. calculate the total stored magnetic energy.



Problem 5

Consider the low energy (~Kev) elastic scattering of a neutron off a target nucleus at rest in the laboratory frame. Let m_1 be the mass of the neutron and m_2 the mass of the nucleus, and $E_1 = \frac{1}{2} m_1 v^2$, the kinetic energy of the neutron.

a. Find the laboratory recoil energy of the nucleus as a function of θ where θ is the recoil angle in the CM system.

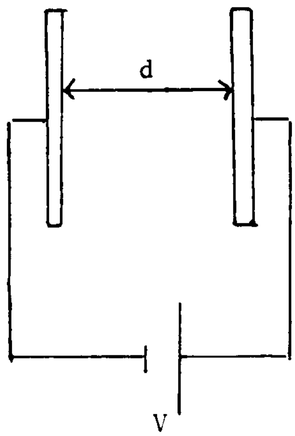
b. If the scattering is isotropic in the CM system, what is the energy distribution, $N(E_R)$, for the recoiling nucleus in the laboratory system? Sketch $N(E_R)$.

Problem 6

A heat engine operates by extracting energy from a container of gas consisting of N atoms with constant specific heat C per atom, initially at temperature T. The engine releases heat into the atmosphere which is at temperature $T_0 < T$. Find the maximum work which can be extracted from the gas.

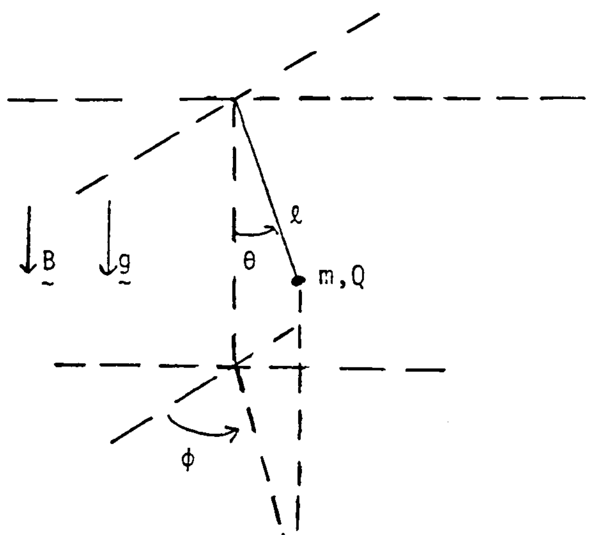
Problem 7

Consider an idealized vacuum diode with voltage drop V. The plate on the left is hot enough to be an excellent electron emitter and the one on the right is an electron absorber. Compute the $I-V$ characteristic of the device, assuming that the current is space-charge limited and neglecting edge effects and thermal velocities.



Problem 8

A spherical pendulum is constructed with a mass m and a massless string of length l (see figure). The configuration of the pendulum is specified by the angles (θ,ϕ) of a spherical coordinate system with the polar axis directed vertically down. Gravity points vertically down and a uniform magnetic field **B** points vertically down. Assuming that the mass carries charge Q construct the Lagrangian and identify constants of the motion. If the mass is released from rest at $(\theta,\phi)=(\frac{\pi}{2},0)$ give the expression for the minimum value of θ reached in the subsequent motion. Evaluate θ_{min} for large and small Q.



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PART II

Problem 9

A spin 1/2 particle interacts with a spatially uniform magnetic field with the Hamiltonian $H = -\mu \vec{\sigma} \cdot \vec{B}$ where $\vec{\sigma}$ are the Pauli-spin matrices.

a. Write out the time dependent Schroedinger equation for the two components of the wavefunction (in the σ_z diagonal basis).

b. Now specialize to the case of time independent B along the $\hat{z}$ axis. Assume the initial state is an eigenstate of $\hat{n} \cdot \vec{\sigma}$, $\hat{n} = \hat{z}\cos\theta + \hat{x}\sin\theta$. Find $\langle \vec{\sigma} \rangle (t)$.

Problem 10

a. What is the average energy $\overline{u}$ of a quantum oscillator of angular frequency ω in thermal equilibrium at temperature T?

b. What is the expectation value for the fluctuations $\dfrac{\overline{(u - \overline{u})^2}}{\overline{u}^2}$?

c. In the Einstein model of a crystal each ion has 3 degrees of freedom with the same frequency ω. Calculate the specific heat of the crystal.

Problem 11

We consider a quantum mechanical potential well of depth V_0 in which a particle of mass m is moving.

a. For the one dimensional case $V = -V_0$ for $|X| < a/2$, $V = 0$ for $|X| > a/2$, find the magnitude of V_0 for which a bound state exists.

b. Do the same for the three dimensional case $V = -V_0$ for $r < a$, $V = 0$ for $r > a$.

c. For the three dimensional case find the differential cross section for scattering of a low energy $\dfrac{2mEa^2}{\hbar^2} \ll 1$ particle.

Problem 12

Consider a system of N particles, each of which can be in one of two quantum states: the ground state with energy 0, and an excited state with energy ε. The system is at temperature T, and $\varepsilon/T \gg 1$ (low temperatures).

a. Find the average number of particles in the excited state assuming the particles are distinguishable.

b. Same as a. assuming the particles are indistinguishable (bosons).

c. In which case are there more particles in the excited state? Explain why.

d. Find the specific heat in both cases. Which one is larger at low temperatures?

Problem 13

The function $f(x)$ is defined by the integral representation

$$f(x) = \frac{1}{2\pi i} \oint_c \exp(z + x/z)\,dz$$

over the contour shown (the unit circle).

Two points for each part.

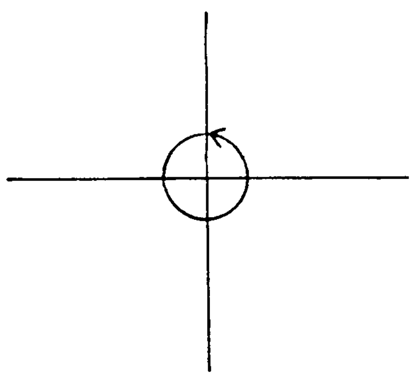
a. Find $f(x)$ for $x \ll 1$.

b. Find the asymptotic form for $f(x)$ with $x \gg 1$.

c. Write a contour integral for $\dfrac{d^2f}{dx^2}$.

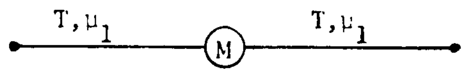
d. Using the result of c. find a second order differential equation for $f(x)$.

e. The solution of this equation can be given in terms of a Bessel function. Using the above results give this solution.



Problem 14

Consider two halves of a string with tension T, mass density μ_1 joined at the origin by a mass M (neglect gravity). Determine the reflection and transmission coefficients for a transverse wave of frequency ω incident from the left. Explain the limiting forms of your answer for high and low frequency.



Problem 15

The anti-matter photon rocket has been perfected. The mass of the fuel is converted with 100% efficiency into photons which are emitted isotropically into the hemisphere behind the rocket.

a. What velocity must the rocket attain if the crew wishes to reach a star 100 light years distant in one year's elapsed time in the rocket frame (neglect time for acceleration). [3 points]

b. If the rocket wishes to deliver a cargo of 100 tons, what mass of fuel must be burned? [7 points]

Problem 16

Consider a dielectric sphere (dielectric constant ε) of radius a.

a. The sphere is immersed in a static electric field which at large distances is given by $E_0\hat{z}$. Calculate $\vec{D}$ and $\vec{E}$ inside the sphere.

b. A long wave length $c/\omega a \gg 1$ electromagnetic wave is incident on the sphere. Using the results of a. calculate the total cross section for scattering.

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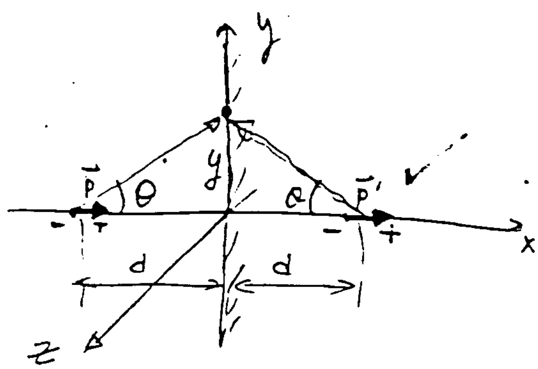
Solution, see the diagram.



the imaginary dipole $\vec{p}' = \vec{p}$

the electric field at $x=0-$ is

$$E = \left(\frac{3p[\hat{x}\cdot(d\hat{x}+y\hat{y})](d\hat{x}+y\hat{y})}{(d^2+y^2)^{5/2}} - \frac{p\hat{x}}{(d^2+y^2)^{3/2}} \right.$$

$$\left. + \frac{3p[\hat{x}\cdot(-d\hat{x}+y\hat{y})](-d\hat{x}+y\hat{y})}{(d^2+y^2)^{5/2}} - \frac{p\hat{x}}{(d^2+y^2)^{3/2}} \right)$$

$$= \left(\frac{6pd^2}{(d^2+y^2)^{5/2}} - \frac{2p}{(d^2+y^2)^{3/2}} \right)\hat{x} = \frac{2p\cos^3\theta}{d^3}(3\cos^2\theta - 1)\hat{x}$$

$\therefore$ charge density $= E/4\pi$.

$\therefore$ force density $f = \dfrac{E^2}{8\pi}(-\hat{x})$

total force $\vec{F} = \int f\, ds$ (see next page)

$$= (-\hat{x})\frac{1}{8\pi}\int E^2\, dy\, dz$$

(assume the length along z direction is L)

$$= \frac{Lp^2}{2\pi d^5}\int_{-\pi/2}^{\pi/2}\cos^6\theta\,(3\cos^2\theta - 1)^2\, d\theta\,(\hat{x})$$

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$$\underline{F} = \int \underline{f}\, dS = \int_0^\infty \frac{E^2}{8\pi} 2\pi \rho\, d\rho\, (-\hat{x}) \cancel{(\rho = d \operatorname{tg}\theta)}$$

where $\rho = d \operatorname{tg} \theta$

$$= (-\hat{x}) \frac{p^2}{d^4} \int_0^{\frac{\pi}{2}} \cos^4\theta\, (3\cos^2\theta - 1)^2 \operatorname{tg}\theta\, d\theta$$

$$= -\frac{3p^2}{8d^4} \hat{x}$$

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1 a) $\approx \sqrt{\frac{GM}{R_e}} \sim 7.9 \text{ km/s}$

2 b) $\mathcal{E} = -\frac{\mu e^4}{2\hbar^2}$, where $\mu = \frac{m_{\mu+} m_{\mu-}}{m_{\mu+} + m_{\mu-}} \simeq \frac{m_\mu}{2} \sim \frac{105 \text{ MeV/c}}{2}$

$\therefore \mathcal{E} \sim 13.6\text{eV} \times \frac{\frac{105}{2} \text{MeV}/c^2}{0.51 \text{ MeV}/c^2} \doteqdot 1428 \text{ eV}$

1 c) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = \ln 2 = \ln 2$

1 d) Spin of D is $S=1$

1 e) n: lifetime τ ~ ~~~~ ~~≃~~ 1000 s.

1 f) $\sim 3R \simeq 6 \text{ cal/mol K}$.

3 g) $\because \frac{R^3}{T^2} = \text{const}$

$\therefore$ period of Saturn is ~~T~~

$1 \text{ year} \times \left(\frac{3}{1}\right)^{3/2} = \sqrt{27} \text{ year} \sim 5.2 \text{ year}$.

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Solution: perturbation, $H'(t) = \begin{cases} 0 & (t<0) \\ -eE_0 z \cdot e^{-t/\tau} & (t>0) \end{cases}$

initially: $\psi_{1s}(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$

wave function:

$$\begin{cases} \psi_{2s} = \frac{1}{4\sqrt{5\pi a_0^3}} e^{-r/2a_0}\left(1-\frac{r}{a_0}\right) \\ \psi_{2p} = \frac{1}{2\sqrt{6a_0^3}} \frac{r}{a_0} e^{-r/2a_0} Y_{1m}(\theta,\phi) \end{cases}$$

$m = 1, 0, -1$

① $P_{2s} = \left| \frac{1}{i\hbar} \int_0^\infty H'_{fi} e^{i\omega_{fi} t} dt \right|^2$

$\left(\text{Here } \omega_{fi} = \frac{-\frac{\frac{1}{2}m\alpha^2c^2}{2^2} - (-\frac{1}{2}m\alpha^2c^2)}{\hbar} = \frac{\frac{3}{4}\times 13.6\,eV}{\hbar} \right)$

$H'_{fi} = \int \psi_{1s}^* (-eEr\cos\theta\, e^{-t/\tau}) \psi_{2s} r^2 dr d\Omega$

selection rule

solid angle

selection rule

$= 0$ Why?

$\therefore P_{2s} = 0$

② $P_{1s\to 2p}$: $\omega_{fi} = \frac{-\frac{\frac{1}{2}mc^2\alpha^2}{2^2} - (-\frac{1}{2}mc^2\alpha^2)}{\hbar^2} = \frac{\frac{3}{4}\times 13.6\,e.}{\hbar}$

$\psi_{2p_0} = \frac{1}{2\sqrt{6a_0^3}} \frac{r}{a_0} e^{-r/2a_0} Y_{10}(\theta\phi)$

(1)

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$$H'_{fi} = \int \psi_{1s}^* \, H'(t) \, \psi_{2p} \, d^3\vec{r}$$ (only $(1s \to 2p)$ $m=0$ is possible)

$$= -\int \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \, eE_0 r\cos\theta \, e^{-t/\tau} \frac{1}{2\sqrt{6\pi a_0^3}} \frac{r}{a_0} e^{-r/2a_0} \, 2\pi r^2 dr \, d(\cos\theta) Y_1$$

$$= -\frac{eE_0 e^{-t/\tau}}{2\sqrt{6}\, a_0^4} \int r^4 e^{-3r/2a_0} dr \cdot (\cos\theta) \, d(\cos\theta) \sqrt{\frac{3}{4\pi}}\cos$$

$$= -\frac{eE_0 e^{-t/\tau}}{2\sqrt{6}\, a_0^4} \cdot \frac{2}{3}\sqrt{\frac{3}{4\pi}} \cdot \int r^4 e^{-3r/2a_0} dr$$

$$= -\frac{eE_0 e^{-t/\tau}}{2\sqrt{6}\, a_0^4} \frac{2}{3}\sqrt{\frac{3}{4\pi}} \cdot 24 \cdot \left(\frac{2}{3}a_0\right)^5$$

$$= -A \, e^{-t/\tau}$$

where $\cancel{A = \frac{2^8 \, eE_0 a_0}{\sqrt{2}\, 3^5 \cdot \sqrt{2\pi}} e^{-t/\tau}}$ — (ii)

$$A = 2^8 eE_0 a_0 / (3^5\sqrt{2})$$

$$P_{1s\to 2p} = \frac{1}{\hbar^2} \left| \int_0^\infty A \, e^{-\frac{t}{\tau} + i\omega t} \right|^2$$ (2)

$$= \frac{A^2}{\hbar^2} \frac{1}{\frac{1}{\tau^2} + \omega^2} = \frac{2^{15} \cdot 3^{-10} (eE_0 a_0)^2}{\left(\frac{\hbar}{\tau}\right)^2 + (\hbar\omega)^2}$$

where $\hbar\omega = \frac{3}{4} \times \frac{1}{2} mc^2 \alpha^2 = \frac{3}{4} \times 13.6\,eV$

Where A and ω are given out by Eqn. (i) & (ii)

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(3) E_0 should be so small that

$$e E_0 a_0 \ll \hbar \omega_{fi}$$

yes

i.e $E_0 \ll \dfrac{\frac{1}{2} m c^2 \alpha^2}{e a_0}$

$\approx$ i.e $E_0 \ll \dfrac{13.6\,eV}{e \cdot 0.53\,\text{Å}} \simeq 27\ V/\text{Å}$

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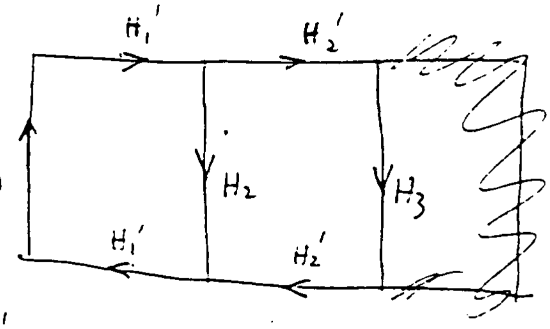
a)



$\nabla \times \vec{H} = \vec{J}$

$\therefore \oint \vec{H} \cdot d\vec{l} = nI$

$$\therefore \begin{cases} H_1 + H_2 + 2H_1' = \frac{nI}{l} \\ 2H_2' + H_3 - H_2 = 0 \end{cases}$$

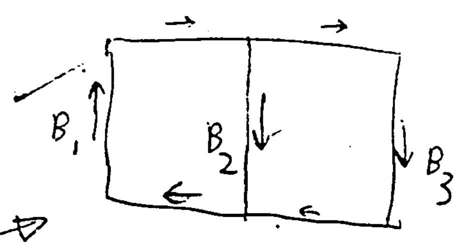
$$\begin{cases} H_1' = H_2 + H_2' \\ H_2' = H_3' \\ H_1' = H_1 \end{cases}$$

$$\Rightarrow \begin{cases} 3H_1 + H_2 = \frac{nI}{l} \\ H_1 = H_2 + H_3 \\ 3H_3 = H_2 . \end{cases} \Rightarrow \begin{cases} H_1 = \frac{4}{15}\frac{nI}{l} . \\ H_2 = \frac{1}{5}\frac{nI}{l} \\ H_3 = \frac{1}{15}\frac{nI}{l} . \end{cases}$$

5

$$\begin{cases} B_1 = \mu_r \mu_0 H_1 = \mu_r \mu_0 \frac{4}{15}\frac{nI}{l} \\ B_2 = \mu_r \mu_0 H_2 = \;\text{"}\;\;\text{"}\;\frac{1}{5}\;\text{"} \\ B_3 = \mu_r \mu_0 H_3 = \;\text{"}\;\;\text{"}\;\frac{1}{15}\;\text{"} . \end{cases}$$

direction of $\vec{B}$ $\longrightarrow$



b) From the result of a) we know that magnetic flux ratio $\frac{\Phi_2}{\Phi_1} = \frac{a^2 B_2}{a^2 B_1} = \frac{3}{4}$

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By Farady law $\mathcal{E} = -\frac{d\phi}{dt}$.

$\therefore \quad \mathcal{E}_2 = -\frac{3}{4}\mathcal{E}$ ✓ 2

c) $\text{energy} = \int \frac{1}{2}\vec{B}\cdot\vec{H}\, dv = \frac{1}{2}\mu_0\mu_r \int H^2 dv$

$= \frac{1}{2}\mu_0\mu_r\left(H_1^2 \cdot 3la^2 + H_2^2 \cdot la^2 + H_3^2\, 3la^2\right)$ ✓

$= \frac{2\mu_0\mu_r}{15l} a^2 n^2 I^2$ 3

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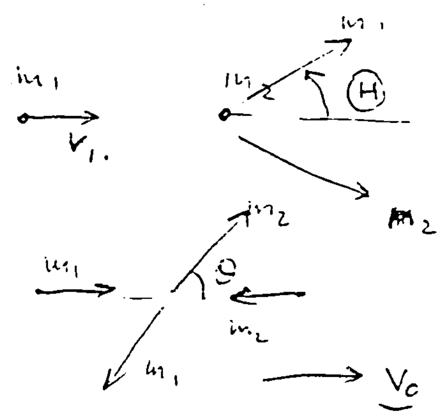

Solution c.o.m. velocity

$$V_c = m_1 V / m_1 + m_2 .$$

Assume V. Θ — are velocity angles in Lab frame

V'. θ — are velocity and angle in CM

Then

$$\begin{cases} V \cos\Theta = V' \cos\theta + V_c & \text{--- (1)} \\ V \sin\Theta = V' \sin\theta \end{cases}$$

The classical approximation of ~~(?)~~ relativistic energy transf. formula gives out

$$E_{Lab} = E_{CM} + \vec{V_c} \cdot \vec{P}'_{CM}$$

in CM: $E_{CM} = \frac{1}{2} m_2 V_c^2$; $\vec{P}'_{CM} \cdot \vec{V_c} = m_2 V_c V_c \cos\theta$

a) $\therefore$ $$E^{(Lab)}_{\text{recoil of } m_2} = \frac{1}{2} m_2 V_c^2 + m_2 V_c^2 \cos\theta$$

$$= \frac{m_2 m_1^2 V_1^2}{2 (m_1 + m_2)^2} (1 + 2\cos\theta)$$

θ — recoil angle in CM

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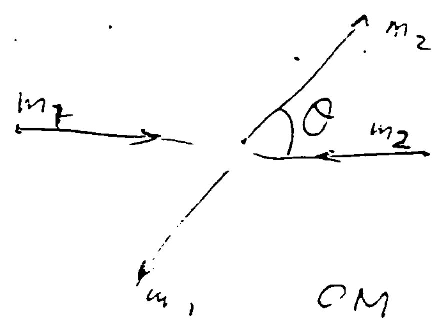
$$= \frac{1}{2} m_2 V_c^2 \,(1 + 2\cos\theta)$$

$$= \frac{m_2 m_1^2 V^2}{2(m_1+m_2)^2}\,(1+2\cos\theta)$$

θ is defined as:



b). $N(E_R)$

$$= \frac{d\mathcal{E}}{d\Omega_{Lab}} = \frac{1}{2\pi}\frac{d\mathcal{E}}{d(\cos\Theta)} = \frac{d\mathcal{E}}{d(\cos\theta)}\,\frac{d(\cos\theta)}{d(\cos\Theta)}\,\frac{1}{2\pi}$$

Solid angle.

by the result of a).

$$\Rightarrow N(E_R) = \frac{1}{2\pi}\cdot\frac{m_2 m_1^2 v_1^2}{(m_1+m_2)^2}\,\frac{d(\cos\theta)}{d(\cos\Theta)}$$

However, By Eqn ① $\Rightarrow$ $\cancel{\tan\Theta = \frac{\sin\theta}{\cos\theta + \frac{V_c}{V}}}$

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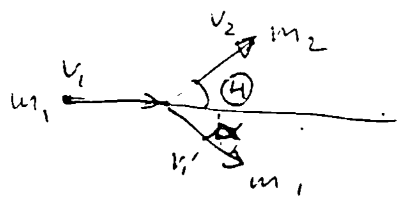
for $m_2 \Rightarrow \sqrt{\frac{2\mathcal{E}}{m_2}}\, d(\cos\Theta) = V' d(\cos\theta) = \frac{m_1 V_1}{m_1+m_2}\, d(\cos\theta)$

i.e. $\frac{d\cos\theta}{d\cos\Theta} = \frac{V_2^{(Lab)}}{V_1}$

$\mathcal{E}$ — energy of m_2 in Lab.

$$\begin{cases} m_1 v_1 = m_2 v_2 \cos\Theta + m_1 v_1' \cos\alpha \\ m_1 v_1' \sin\alpha = m_2 v_2 \sin\Theta \end{cases}$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_1 v_1'^2$$



$$\Rightarrow V_2^{\mathcal{E}}(\Theta) = \frac{2m_1 v_1}{m_1+m_2}\cos\Theta$$

$$\therefore \mathcal{E}(\Theta) = \frac{2m_1^2 v_1^2 m_2}{(m_1+m_2)^2}\cos^2\Theta \Rightarrow \cos\Theta = \frac{(m_1+m_2)\sqrt{E_k}}{m_1 v_1 \sqrt{2m_2}}$$

$$\therefore \frac{d(\cos\theta)}{d(\cos\Theta)} = \frac{\frac{2m_1 v_1}{m_1+m_2}\cos\Theta}{\frac{m_1 v_1}{m_1+m_2}} = 2\cos\Theta$$

$$\therefore N(E_k) = \cancel{4}\,\frac{m_2 m_1^2 v_1^2}{\pi(m_1+m_2)^2}\cos\Theta = \frac{m_1 m_2 V_1}{\pi(m_1+m_2)}\sqrt{\frac{E_k}{2m_2}}$$

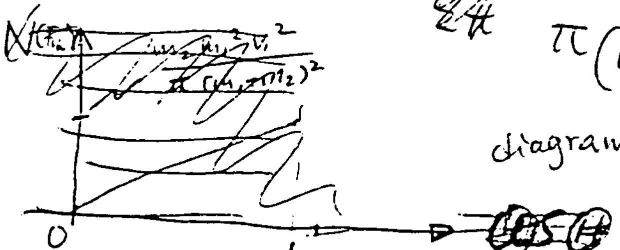
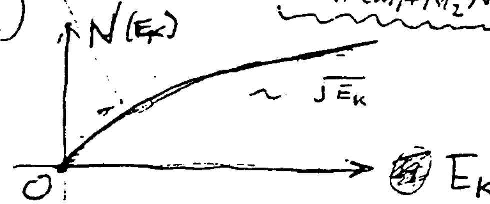


diagram →



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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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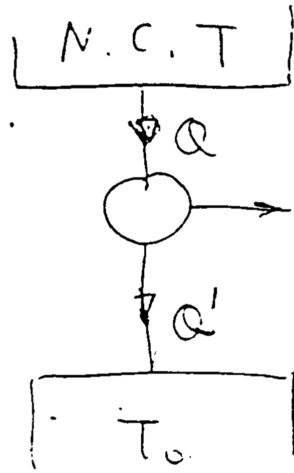
Solution: maximum work occurs

when total entropy change is zero



$$\Delta S_{total} = 0$$

i.e $\displaystyle\int_T^{T_o} \frac{NC\,dT}{T}$ (ΔS Entropy Change of gas in container) $T > T_o$

$+ \displaystyle\oint \frac{Q'}{T_o} = 0$ (Q' is heat released to atmosphere)

(entropy change of atmosphere)

$$\therefore Q' = T_o NC \ln \frac{T}{T_o}$$

$$\therefore W_{max} = Q - Q' = \int_{T_o}^{T} \frac{CN\,dT}{\cancel{CN}} - Q'$$

$$= \cancel{CN}\, NC\,(T - T_o) - NC\,T_o \ln \frac{T}{T_o}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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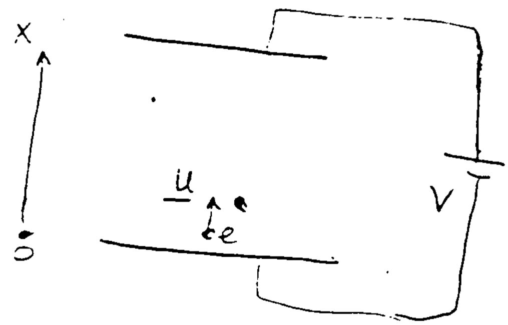
Solution: charge $q = -e$



assume the electric potential is $\phi(x)$

then

$$\begin{cases} \nabla \cdot \underline{E} = \dfrac{\rho}{\varepsilon_0} \quad \& \quad \underline{E} = \nabla\phi \\ J = \rho u \end{cases}$$

ρ charge density

J current density

u velocity of electron

$$\Rightarrow \quad -\nabla^2\phi = J/\varepsilon_0 u$$

However, energy conservation $\Rightarrow$

$$-q\phi = \frac{1}{2}mu^2 \Rightarrow u = \sqrt{\frac{-2q\phi}{m}}$$

$$-\frac{d^2\phi}{dx^2} = \frac{J}{\epsilon_0\sqrt{\frac{-2q}{m}}}\phi^{-1/2}$$

Assume

$$\begin{cases} \dfrac{d\phi}{dx}\Big|_{x=0} = 0 \quad (\because E|_{x=0} = 0) \\ \phi|_{x=d} = V; \quad \phi'_{x=0} = 0 \end{cases}$$

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we have $-\frac{1}{2}\cdot\frac{d\left(\frac{d\phi}{dx}\right)^2}{d\phi} = \frac{J}{\epsilon_0\sqrt{\frac{-2e}{m}}}\,\phi^{-1/2}$

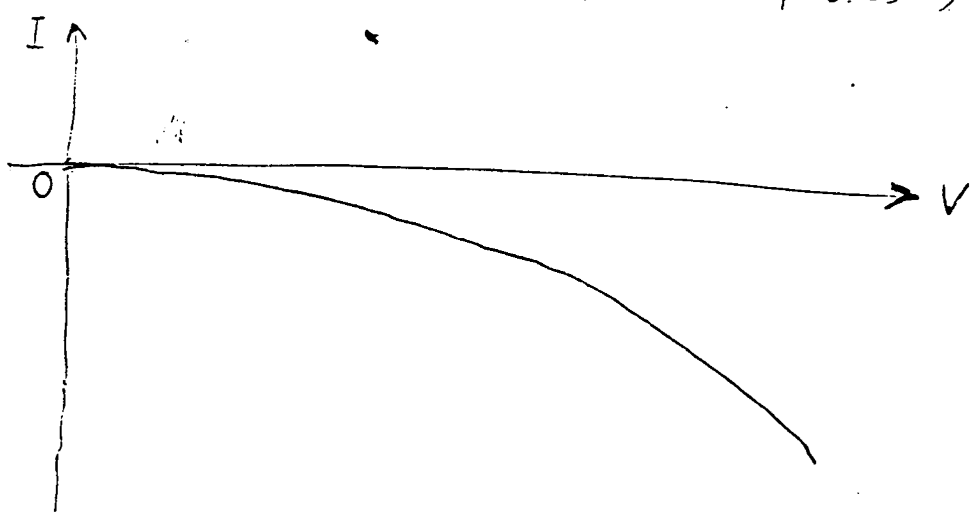
$\Rightarrow \phi(x)^{\frac{3}{4}} = \frac{3}{4}\sqrt{\frac{-4J}{\epsilon_0\sqrt{\frac{-2e}{m}}}}\; x$

$(q=-e)$

$\phi\big|_{x=d} = V \Rightarrow J = \frac{+4\,\epsilon_0}{9d^2}\sqrt{\frac{2e}{m}}\; V^{3/2}$

$\therefore\; I = A\cdot J = \frac{-4A}{9d^2}\sqrt{\frac{2e}{m}}\; V^{3/2}$

(A is the area of the plates)



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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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① i Lagrangian

$$L = T - V_1 + \frac{Q}{c}\vec{A}\cdot\vec{v}$$

$$= \frac{m}{2}\ell^2(\dot\theta^2 + \sin^2\theta\,\dot\phi^2) + mg\ell\cos\theta - \frac{QB\ell^2}{2c}\sin^2\theta\,\dot\phi$$

② $\frac{d}{dt}\frac{\partial L}{\partial\dot\phi} - \frac{\partial L}{\partial\phi} = 0 \Rightarrow P_\phi = \frac{\partial L}{\partial\dot\phi} = \text{constant}.$

i/ i.e $P_\phi = m\ell^2\sin^2\theta\,\dot\phi - \frac{QB\ell^2}{2c}\sin^2\theta = \cancel{Q}(\text{const})$

$\because \frac{\partial L}{\partial t} = 0 \quad \therefore \quad H = \text{Const}.$

ii/ ~~However~~

$$\therefore H = \frac{\partial L}{\partial\dot\theta}\dot\theta + \frac{\partial L}{\partial\dot\phi}\dot\phi - L$$

$$= \frac{1}{2}m\ell^2\dot\theta^2 + \frac{\left(P_\phi + \frac{QB\ell^2}{2c}\sin^2\theta\right)^2}{2m\ell^2\sin^2\theta} - mg\ell\cos\theta$$

$$= E = \text{const}$$

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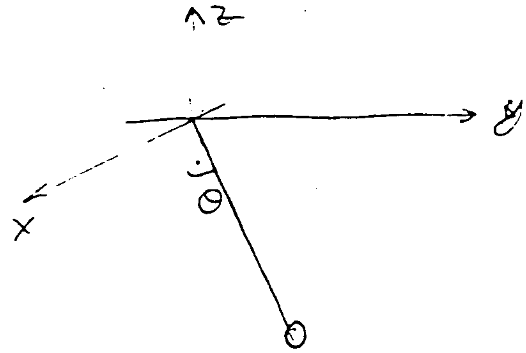
Solution: see diagram.



$$\begin{cases} x = l \sin\theta \cos\phi \\ y = l \sin\theta \sin\phi \\ z = -l \cos\theta \end{cases}$$

∴ kinetic energy

$$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = \frac{1}{2} m \left(l^2 \dot{\theta}^2 + l^2 \sin^2\theta \, \dot{\phi}^2 \right)$$

gravity potential energy

$$V_1 = mgz = -mgl\cos\theta$$

$\vec{B} = -B\,\vec{e}_z$. ∴ we can choose vector potential

$$\vec{A} = \frac{1}{2} \vec{B} \times \vec{r} = -\frac{B}{2} \vec{e}_z \times \vec{r}$$

$$= \frac{Bl}{2} \sin\theta \left(\sin\phi \, \vec{e}_x - \cos\phi \, \vec{e}_y \right) = -\frac{Bl}{2} \sin\theta$$

(Next page)

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(3) By the initial condition $(\theta, \phi) = (\frac{\pi}{2}, 0)$

we have constants: $(\dot\theta\ \dot\phi) = (0, 0)$.

$$\begin{cases} P_\phi = -\dfrac{QB\ell^2}{2c} = m\ell^2\sin^2\theta\,\dot\phi - \dfrac{QB\ell^2}{2c}\sin^2\theta \\ E = 0 \end{cases}$$

put $P_\phi = -\dfrac{QB\ell^2}{2c}$ in $E=0$

we have

$$E = 0 = \frac{1}{2}m\ell^2\dot\theta^2 + \frac{\left(\frac{QB\ell^2}{2c}\right)^2\cos^4\theta}{2m\ell^2\sin^2\theta} - mg\ell\cos\theta$$

Letting $\dot\theta = 0 \Rightarrow \theta_{min}$:

$$0 = \frac{Q^2B^2\ell^2\cos^4\theta}{8mc^2\sin^2\theta} - mg\ell\cos\theta$$

$$\Rightarrow \frac{Q^2B\ell}{8m^2c^2g}\cos^3\theta + \cos^2\theta - 1 = 0.$$

i/. large Q.

$$\frac{Q^2B\ell}{8m^2c^2g} \gg 1 \Rightarrow \frac{Q^2B\ell}{8m^2c^2g}\cos^3\theta_m - 1 \simeq 0$$

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Then $\theta_{min} \sim \cos^{-1} \left[\frac{8 m^2 c^2 g}{Q^2 B l} \right]^{1/3}$.

ii/. Q very small

Then $\cos^2\theta_{min} \sim 1$. $\therefore \theta_{min} \simeq 0$.

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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a) Assume $\vec{B} = B(\cos\theta\ \vec{e}_z + \sin\theta\cos\phi\ \vec{e}_x + \sin\theta\sin\phi\ \vec{e}_z)$

wavefunction $|\psi\rangle = a|+\rangle + b|-\rangle$: $\begin{pmatrix} a \\ b \end{pmatrix}$

Then $\hat{H} = -\mu B\ \vec{\sigma}\cdot\vec{n}$

$$= -\mu B\left[\cos\theta\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \sin\theta\cos\phi\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \sin\theta\sin\phi\cdot\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right]$$

$$= -\mu B\begin{bmatrix} \cos\theta & \sin\theta\, e^{-i\phi} \\ \sin\theta\, e^{i\phi} & -\cos\theta \end{bmatrix}$$

∴ Schrödinger Eqn is $i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle$

$$\Rightarrow\quad i\hbar\begin{bmatrix} \dot{a} \\ \dot{b} \end{bmatrix} = -\mu B\begin{bmatrix} \cos\theta & \sin\theta\, e^{-i\phi} \\ \sin\theta\, e^{i\phi} & -\cos\theta \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

i.e. $i\hbar\dot{a} = -\mu B(\cos\theta\ a + \sin\theta\ e^{-i\phi}\ b)$

$i\hbar\dot{b} = -\mu B(\sin\theta\ e^{i\phi}\ a - \cos\theta\ b)$

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b) Now $\vec{B} = B\,\vec{e}_z$.

Assume the eigenstate of $\hat{n}\cdot\vec{\sigma}$ is $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = |\varphi\rangle$

Then

$$\left[\cos\theta \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \sin\theta \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right]\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \sigma_n \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\Rightarrow \begin{cases} \sigma_n = 1 \ ; & |\varphi_+\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix} \\ \sigma_n = -1 \ ; & |\varphi_-\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ -\sin\frac{\theta}{2} \end{pmatrix} \end{cases}$$

Assume initial state to be ~~$|\psi$~~ $|\psi(0)\rangle = |\varphi_+\rangle$

$$|\psi(0)\rangle = \cos\frac{\theta}{2}\,|+\rangle + \sin\frac{\theta}{2}\,|-\rangle .$$

Then $$|\psi(t)\rangle = \cos\frac{\theta}{2}\, e^{-iE_+t/\hbar}\,|+\rangle + \sin\frac{\theta}{2}\, e^{-iE_-t/\hbar}\,|-\rangle$$

where $E_+ = -\mu B$, $E_- = \mu B$

$$\therefore |\psi(t)\rangle = \cos\frac{\theta}{2}\, e^{i\mu Bt/\hbar}\,|+\rangle + \sin\frac{\theta}{2}\, e^{-i\mu Bt/\hbar}\,|-\rangle$$

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$$\therefore \langle \sigma_z \rangle_t = \left| \cos\frac{\theta}{2}\, e^{i\mu B t/\hbar} \right|^2 - \left| \sin\frac{\theta}{2}\, e^{-i\mu B t/\hbar} \right|^2 = \cos\theta$$

$$\langle \sigma_x \rangle_t = \langle \psi(t) | \sigma_z | \psi(t) \rangle$$

$$= \left(\cos\frac{\theta}{2}\, e^{-i\mu B t/\hbar} \quad \sin\frac{\theta}{2}\, e^{i\mu B t/\hbar} \right) \begin{pmatrix} \sin\frac{\theta}{2}\, e^{-i\mu B t/\hbar} \\ \cos\frac{\theta}{2}\, e^{i\mu B t/\hbar} \end{pmatrix}$$

$$= \sin\theta \cos\left(2\mu B t/\hbar\right)$$

$$\langle \sigma_y \rangle_t = \left(\cos\frac{\theta}{2}\, e^{-i\mu B t/\hbar} \quad \sin\frac{\theta}{2}\, e^{i\mu B t/\hbar} \right) \begin{pmatrix} -i \sin\frac{\theta}{2}\, e^{-i\mu B t/\hbar} \\ i \cos\frac{\theta}{2}\, e^{i\mu B t/\hbar} \end{pmatrix}$$

$$= - \sin\theta \sin\frac{2\mu B t}{\hbar}$$

$$\therefore \langle \vec{\sigma} \rangle_t = \cos\theta\, \vec{e}_z + \sin\theta \cos\frac{2\mu B}{\hbar} t\, \vec{e}_x - \sin\theta \sin\frac{2\mu B t}{\hbar}\, \vec{e}_y$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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a) The partition function is $\left(\beta = \frac{1}{kT}\right)$

$$Z = \sum_{n=0}^{\infty} e^{-\beta(n+\frac{1}{2})\hbar\omega} = \left(e^{\frac{\beta\hbar\omega}{2}} - e^{-\beta\frac{\hbar\omega}{2}}\right)^{-1}$$

$$\therefore \bar{u} = -\frac{\partial}{\partial\beta}\ln Z = \frac{\frac{\hbar\omega}{2}\left(e^{\beta\frac{\hbar\omega}{2}} + e^{-\beta\frac{\hbar\omega}{2}}\right)}{e^{\frac{\beta\hbar\omega}{2}} - e^{-\beta\frac{\hbar\omega}{2}}} = \frac{\hbar\omega}{2}\coth\frac{\beta\hbar\omega}{2}$$

b) $$\overline{(u-\bar{u})^2} = \overline{u^2} - \bar{u}^2$$

$$\overline{u^2} = \frac{\sum_{n=0}^{\infty}\left[(n+\frac{1}{2})\hbar\omega\right]^2 e^{-\beta(n+\frac{1}{2})\hbar\omega}}{Z}$$

however

$$\sum_{n=0}^{\infty}\left[(n+\tfrac{1}{2})\hbar\omega\right]^2 e^{-\beta(n+\frac{1}{2})\hbar\omega} = -\frac{\partial}{\partial\beta}\sum_{n=0}^{\infty}(n+\tfrac{1}{2})\hbar\omega\, e^{-\beta(n+\frac{1}{2})\hbar\omega}$$

$$= \frac{\partial}{\partial\beta}\left[\frac{\partial}{\partial\beta}\sum_{n=0}^{\infty} e^{-\beta(n+\frac{1}{2})\hbar\omega}\right]$$

$$\therefore \overline{u^2} - \bar{u}^2 = -\frac{\partial\bar{u}}{\partial\beta} = -\frac{\hbar\omega}{2}\frac{\partial}{\partial\beta}\left(\coth\frac{\beta\hbar\omega}{2}\right)$$

$$= \frac{(\hbar\omega)^2}{\left(e^{\frac{\beta\hbar\omega}{2}} - e^{-\frac{\beta\hbar\omega}{2}}\right)^2}$$

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$$\therefore \quad \frac{\overline{u^2} - \bar{u}^2}{\bar{u}^2} = 4 \Big/ \left(e^{\frac{\hbar\omega}{2kT}} + e^{-\frac{\hbar\omega}{2kT}}\right)^2 = \operatorname{sech}^2 \frac{\hbar\omega}{2kT}$$

c) Consider N ions' crystal

$$C_v = N \times 3 \times \frac{\partial \bar{u}}{\partial T}$$

$$= 3Nk\left(\frac{\hbar\omega}{kT}\right)^2 \Big/ \left(e^{\frac{\hbar\omega}{2kT}} - e^{-\frac{\hbar\omega}{2kT}}\right)^2$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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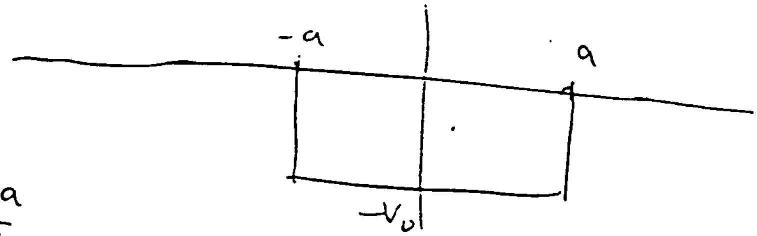
a) 1-D,



Assume

$$\psi = \begin{cases} A\, e^{-kx} & x > \frac{a}{2} \\ B \cos k'x & -\frac{a}{2} \le x \le \frac{a}{2} \\ A\, e^{kx} & x < -\frac{a}{2} \end{cases}$$

✓

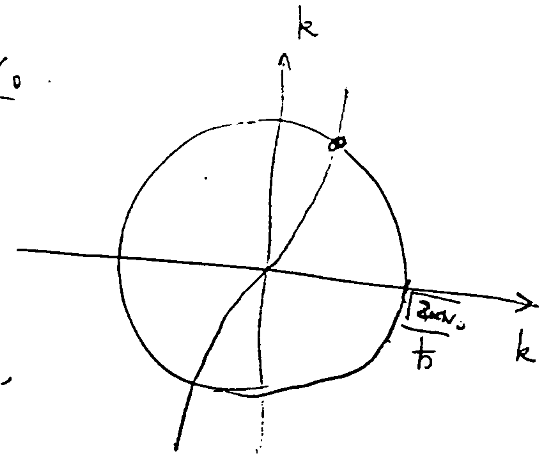
$$k = \frac{\sqrt{-2mE}}{\hbar}$$

$$k' = \frac{\sqrt{2m(V_0+E)}}{\hbar}$$

ψ and ψ' continuous at $x = \frac{a}{2}$ $\Rightarrow$

$$k = k' \operatorname{tg} k' \frac{a}{2}.$$

However $k^2 + k'^2 = \dfrac{2mV_0}{\hbar^2}$.



From the diagram, we know that as long as $V_0 > 0$, There is a bound state solution for (k & k'),

i.e. for 1 dimension. V_0 should : $V_0 > 0$ for which a bound state exists ✓

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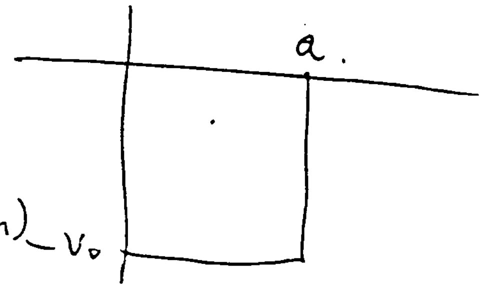
b). 3-D:



Let $\chi = r R(r)$

($R(r)$ is the radial wavefunction)

for s-wave. ($l=0$)

Then

$$\begin{cases} \dfrac{d^2\chi}{dr^2} + \dfrac{2m}{\hbar^2}(E+V_0)\chi = 0. & 0<r<a \\ \chi'' + \dfrac{2mE}{\hbar^2}\chi = 0 & r>a \end{cases}$$

Assume

$$\chi = \begin{cases} A \sin kr & ; \quad k = \dfrac{\sqrt{2m(E+V_0)}}{\hbar} \\ B\, e^{-k'r} & ; \quad k' = \dfrac{\sqrt{\cancel{\ } 2mE}}{\hbar} \end{cases}$$

Then $\frac{d}{dr}(\ln\chi)$ continuous at $r=a$ $\Rightarrow$

$$k' = -k \operatorname{ctg} ka$$ ✓

Let $ka = \eta$. $k'a = \xi$. Then

$$\begin{cases} \eta^2 + \xi^2 = \dfrac{2mV_0}{\hbar^2}a^2 \\ \xi = -\eta \operatorname{ctg} \eta. \end{cases}$$

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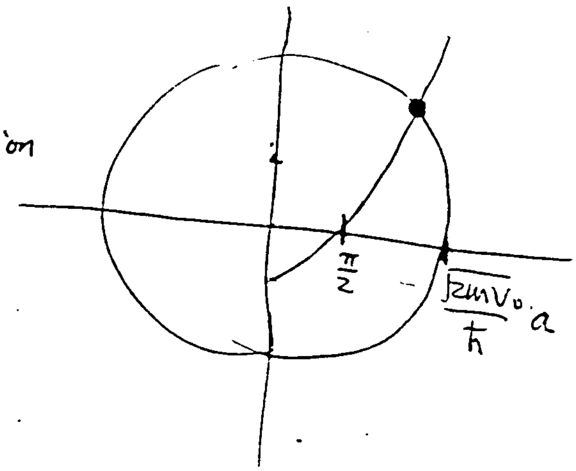
see diagram



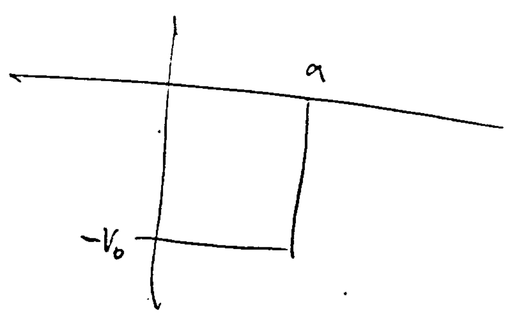
In order to have a solution of $\xi > 0, \eta > 0$ we should require

$$\frac{\pi}{2} \le \frac{2mV_0}{\hbar}$$

i.e. $V_0 \geq \frac{\pi^2\hbar^2}{8ma^2}$. ✓

:) . $\because \frac{2mEa^2}{\hbar^2} \ll 1$ ∴ we can consider only s wave ✓

Using the notation in b).



Assume $\chi = \begin{cases} A \sin k_1 r & 0 < r < a \\ B \sin(k_2 r + \delta) . & r > a . \end{cases}$

$\begin{cases} k_1 = \frac{\sqrt{2m(E+V_0)}}{\hbar} \\ k_2 = \frac{\sqrt{2mE}}{\hbar} \end{cases}$

Then, $(\ln \chi)'$ continuous at $r = a \Rightarrow$

$$k_1 \operatorname{ctg} k_1 a = k_2 \operatorname{ctg}(k_2 a + \delta)$$

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$$\therefore \delta = tg^{-1}\left(\frac{k_2}{k_1} tg\, k_1 a\right) - k_2 a \quad \checkmark$$

$$\therefore \frac{d\sigma}{d\Omega} = \frac{4\pi}{k_2^2} \sin^2 \delta$$

[illegible] of 4π

$$= \frac{2\pi \hbar^2}{\cancel{2} m E} \sin^2 \left[tg^{-1}\left(\sqrt{\frac{E}{E+V_0}}\, tg\left(\frac{\sqrt{2m(E+V_0)}}{\hbar} a\right) - \frac{\sqrt{2mE}}{\hbar} a \right)\right]$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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a). in the excited state.

$$N_e^{(a)} = N_{excited} = N \cdot \frac{e^{-\beta \cdot \varepsilon}}{e^{-\beta 0} + e^{-\beta\varepsilon}}$$

$$= \frac{N e^{-\beta\varepsilon}}{1 + e^{-\beta\varepsilon}} = \frac{N}{e^{\beta\varepsilon}+1} \sim \underline{N e^{-\beta\varepsilon}}.$$

b) in excited state

$$N_e^{(b)} = N_{excited} = \frac{N e^{-\beta N\varepsilon} + (N-1) e^{-\beta(N-1)} + \cdots + 1 \cdot e^{-\beta\varepsilon} + 0}{e^{-\beta N\varepsilon} + e^{-\beta(N-1)\varepsilon} + \cdots + e^{-\beta\varepsilon} + 1}$$

$$= -\frac{1}{\varepsilon}\frac{\partial}{\partial\beta} \ln\left(1 + e^{-\beta\varepsilon} + \cdots + e^{-\beta(N-1)\varepsilon} + e^{-\beta\varepsilon N}\right)$$

$$= \frac{N+1}{e^{\beta(N+1)\varepsilon} - 1} - \frac{1}{e^{\beta\varepsilon} - 1}$$

$$\sim (N+1)\, e^{-\beta(N+1)\varepsilon} - e^{-\beta\varepsilon}.$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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c) in a). There are more particles in excited state.

The reason is that Bosons always statistically try to come together — to $\varepsilon = 0$ state.

d). $C_v^{(a)} = \frac{\partial}{\partial T}(N_e^{(a)}\varepsilon) = Nk\left(\frac{\varepsilon}{kT}\right)^2 e^{\frac{\varepsilon}{kT}} \Big/ \left(1+e^{\frac{\varepsilon}{kT}}\right)^2$

$$C_v^{(b)} = \frac{\partial}{\partial T}\left(N_e^{(b)}\varepsilon\right)$$

$$= \frac{k\left(\frac{(N+1)\varepsilon}{kT}\right)^2}{\left(e^{\beta(N+1)\varepsilon}-1\right)^2} - \frac{k\left(\frac{\varepsilon}{kT}\right)^2}{\left(e^{\beta\varepsilon}-1\right)^2}$$

at low T: $C_v^{(b)} > C_v^{(a)}$ because of the Bose-Einstein Condensation.

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Please insert on each page the Problem No. 13 and your Identification No. 34

Score = 3

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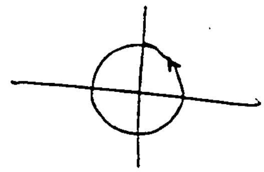
a)
$$f(x) = \frac{1}{2\pi i} \oint e^{z+\frac{x}{z}} \, dz.$$

for $x \ll 1$.



$$e^{z+\frac{x}{z}} = e^{z}\left(1 + \frac{x}{z} + \frac{x^2}{2!\,z^2} + \cdots\right)$$

$$\therefore f(x) \simeq \frac{1}{2\pi i} \oint \frac{x e^{z}}{z} \, dz.$$

$$= x \cdot \mathrm{Res}\left(\frac{e^z}{z}\right)\Big|_{z=0} = x$$

$x \ll 1$: $f(x) \simeq x$

b) ~~for $x \gg 1$. Let $g(z) = z + \frac{x}{z}$.~~

~~$g'(z)\big|_{z_0} = 0 \Rightarrow z_0 = \sqrt{x}\, e^{i\phi}$~~

~~$g''(z_0) = 2\sqrt{x}\, e^{-3i\phi} \Rightarrow$ choose $\phi = \frac{\pi}{3}$~~ $\Rightarrow \phi_0 = \sqrt[4]{x}\, e^{i\frac{\pi}{3}}$

~~Then $g(z_0+s) \simeq g(z_0) + \frac{1}{2} g''(z_0)\, s^2$.~~

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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b) $x \gg 1$. $z = e^{i\phi}$.

$$\therefore \quad z + \frac{x}{z} = e^{i\phi} + x e^{-i\phi} \sim -i x \sin\phi.$$

$$f(x) \simeq \frac{1}{2\pi} \int_0^{2\pi} e^{i(\phi - x\sin\phi)} \, d\phi.$$

Let $g(\phi) = \phi - x\sin\phi$ steepest method.

$g'(\phi_0) = 1 - x\cos\phi = 0 \Rightarrow \phi_0 = \cos^{-1}\frac{1}{x}$

$g''(\phi_0) = x\sin\phi_0$. Choose phase of ϕ_0 so that $g''(\phi_0) < 0$

we have $f(x) \simeq \sqrt{\frac{2\pi}{x}}$.

c). $$\frac{d^2 f}{dx^2} = \frac{1}{2\pi i} \oint \frac{e^{(z + \frac{x}{z})}}{z^2} \, dz.$$

d). $$\frac{d^2 f}{dx^2} + \frac{1}{x}\frac{df}{dx} + \left(1 - \frac{1}{x^2}\right) f(x) = 0$$ Why?

$$f'' + \frac{1}{x} f' + \left(1 - \frac{1}{x^2}\right) f = 0.$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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e) This solution is $J_1(x)$.

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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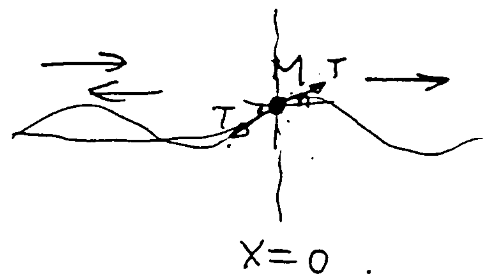
Assume that M is at $x=0$



$x<0$: $\psi_- = e^{i(kx-\omega t)} + r e^{-i(kx-\omega t)}$

$x>0$: $\psi_+ = t\, e^{i(kx-\omega t)}$

at $x=0$:

$$\begin{cases} T\left(\dfrac{\partial \psi_+}{\partial x} - \dfrac{\partial \psi_-}{\partial x}\right) = M \dfrac{\partial^2 \psi}{\partial t^2} & \text{at } x=0 \\ \psi_+ = \psi_- & \text{at } x=0. \end{cases}$$

$$\Rightarrow \begin{cases} 1+r=t. \\ T(ikt - ik + ikr) = -M\omega^2 t \end{cases}$$

$$\Rightarrow \begin{cases} t = \dfrac{2}{2+\dfrac{M\omega^2}{ikT}} = \dfrac{2ikT}{M\omega^2+2ikT}. \\ r = \dfrac{M\omega^2/ikT}{2+\dfrac{M\omega^2}{ikT}} = \dfrac{M\omega^2}{M\omega^2+2ikT} \end{cases}$$

Where $k = \omega/\sqrt{\frac{T}{\mu_1}} = \omega\sqrt{\frac{\mu_1}{T}}$.

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i) high ω:
$$\begin{cases} t \sim \dfrac{2kT}{M\omega^2}\, i \sim 0. \\ r \sim 1 \end{cases}$$

ii) low ω:
$$\begin{cases} t \sim 1 \\ r \sim \dfrac{M\omega^2}{2ikT} \sim 0. \end{cases}$$

~~explain: for ω high ω~~

−1

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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Solution.

a. Assume the velocity is v

$$\frac{vt}{\sqrt{1-v^2/c^2}} = r$$

$$\therefore \frac{v/c}{\sqrt{1-v^2/c^2}} = \frac{r}{ct} = 100 \qquad (t = 1\,\text{year})$$

$$\therefore v = c\sqrt{\frac{10^4}{10^4+1}} \doteq 0.99995c \checkmark$$

b. $$\int_0^{\frac{\pi}{2}} \frac{\hbar\omega}{c}\cos\theta \cdot 2\pi\sin\theta\, d\theta \Big/ 4\pi\frac{\hbar\omega}{c} = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1}{2}\sin 2\theta\, d\theta$$

$$= \frac{1}{4}$$

neglect the mass of the rocket.

initial $M = M_{cargo} + M_{fuel}$ $\quad \Sigma P = 0$

final : fuel $\rightarrow$ energy $M_f c^2$

$\rightarrow$ momentum $-\frac{1}{4}M_f c$

Cargo $\rightarrow$ momentum $= \frac{1}{4}M_f c$

$$\therefore \frac{M_{cargo} V}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{1}{4}M_f c$$

(conservation of momentum)

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$$\therefore M_{fuel} = 4 M_{cargo} \frac{v/c}{\sqrt{1 - v^2/c^2}}$$

$$= 400 M_{cargo}$$

$$= 4 \times 10^4 \text{ tons}$$

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PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

Score = 10

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the **Problem No.** 16 and your **Identification No.** 34.

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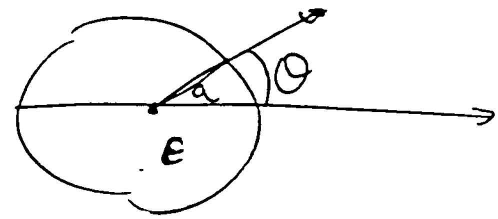
a) $\vec{E} = -\nabla\phi.$



$\nabla^2\phi = -4\pi\rho.$

The external field will induced the dipole $\vec{p}$. $\longrightarrow \vec{E}$

Assume :

$$\begin{cases} r>a: & \phi_o = -\vec{E}_0\cdot\vec{r} + \dfrac{\vec{p}\cdot\vec{r}}{r^3}. \\ r<a: & \phi_i = \sum_{\ell=0}^{\infty} a_\ell r^\ell P_\ell(\cos\theta) \end{cases}$$

Boundary ~~Conda~~ Condition:

$$\begin{cases} \phi_o(r=a) = \phi_i(r=a) \\ \left.\dfrac{\partial\phi_o}{\partial r}\right|_{r=a} = \epsilon \left.\dfrac{\partial\phi_i}{\partial r}\right|_{r=a} \end{cases}$$

$$\Rightarrow \begin{cases} \phi_o = -\vec{E}_0\cdot\vec{r} + \dfrac{\vec{E}_0\cdot\vec{r}}{r^3}\,\dfrac{\varepsilon-1}{\varepsilon+2}\,a^3. \\ \phi_i = -\dfrac{3}{\epsilon+2}\,\vec{E}_0\cdot\vec{r} \end{cases}$$

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is inside the sphere.

$$\begin{cases} \underline{E} = -\nabla\phi_i = \dfrac{3}{\epsilon+2}\,\vec{E}_0 \\[2ex] \underline{D} = \epsilon\,\underline{E} = \dfrac{3\epsilon}{\epsilon+2}\,\underline{E}_0 \end{cases}$$

b) From the result of a. we have induced dipole ϕ_0 in

$$\vec{p} = \frac{\epsilon-1}{\epsilon+2}\,a^3\,\underline{E}_0$$

$\therefore$ ~~$\frac{d\sigma}{d\Omega}$~~ $\underline{H} = i\,k\,\underline{n} \times \dfrac{e^{ikR}}{R}\,\dfrac{\epsilon-1}{\epsilon+2}\,a^3\,(-i\omega\underline{E}_0)$

$$\underline{E} = \underline{H} \times \underline{n}$$

$$\therefore \frac{d\sigma}{d\Omega} = R^2\,\frac{c}{4\pi}\,|\underline{E}\times\underline{H}| \Big/ \frac{c}{4\pi}\,|E_0 \times \underline{H}_0|$$

and $\sigma_{total} = \displaystyle\int \frac{d\sigma}{d\Omega}\cdot d\Omega$ = (next page.)

PHYSICS DEPARTMENTAL WRITTEN EXAMINATION

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$$= \frac{8\pi}{3}\left(\frac{\epsilon-1}{\epsilon+2}\right)^2\left(\frac{\omega}{c}\right)^4 \cdot a^2 \cdot$$

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$$I_{incident} = \frac{c}{4\pi} E_0^2$$

$$\therefore \sigma_{tot} = \frac{P_{tot}}{I} = \frac{8\pi}{3c^4}\left(\frac{\varepsilon-1}{\varepsilon+2}\right)^2 a^6 \omega^4$$

$$\therefore \boxed{\sigma_{tot} = \frac{8\pi\,\omega^4 a^6}{3c^4}\left(\frac{\varepsilon-1}{\varepsilon+2}\right)^2}$$

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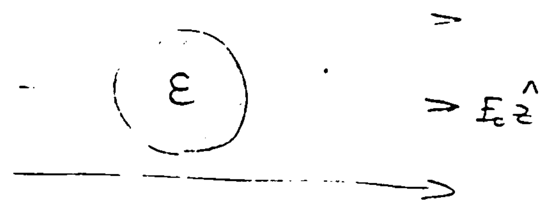
Solution.

a. Assume $\vec{P}$ = P_z inside sphere, the "anti-electric" field is



$$E' = -\frac{4\pi}{3}P$$

$$\therefore \begin{cases} E = E_0 + E' = E_0 - \frac{4\pi}{3}P \\ D = \varepsilon E = E + 4\pi P \end{cases}$$

$$\Longrightarrow \quad E = \frac{3}{\varepsilon+2}E_0 \qquad D = \frac{3\varepsilon}{\varepsilon+2}E_0 \qquad \vec{P} = \frac{3}{4\pi}\frac{\varepsilon-1}{\varepsilon+2}E_0\hat{z}$$

∴ In-side the sphere

$$\vec{E} = \frac{3}{\varepsilon+2}E_0\hat{z} \quad ✓, \quad \vec{D} = \frac{3\varepsilon}{\varepsilon+2}E_0\hat{z} \quad ✓$$

b. for $c/\omega a \gg 1$, we can consider the field across the sphere to be ~~than~~ uniform.
from a, we have the dipole

$$\vec{d} = \frac{4\pi}{3}a^3\vec{P} = \frac{\varepsilon-1}{\varepsilon+2}E_0 a^3\hat{z}$$

$$|\ddot{\vec{d}}| = \left|\omega^2\frac{\varepsilon-1}{\varepsilon+2}E_0 a^3\right| \hat{z}$$

$$P_{tot} = \frac{2}{3c^3}|\ddot{\vec{d}}|^2 = \frac{2}{3c^3}\left(\frac{\varepsilon-1}{\varepsilon+2}\right)^2 E_0^2 a^6 \omega^4$$

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